Chinese Journal of Physics 54 (2016) 352–364

Contents lists available at ScienceDirect

Chinese Journal of Physics

journal homepage: http://www.journals.elsevier.com/chinese-journal-of-physics/

Dynamics of modulated waves in a dissipative nonlinear network with nonlinear dispersion

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ARTICLE INFO

Article history:
Received 17 February 2016
Received in revised form 7 April 2016
Accepted 8 April 2016
Available online 3 May 2016

ABSTRACT

In this present paper, we investigate analytically the dynamics of modulated waves in a dissipative nonlinear network with nonlinear dispersion. In the small amplitude limit, we use the reductive perturbation method and the continuum limit approximation to derive an envelope Ginzburg-Landau (GL) equation of the network. Considering modulated Stokes waves propagating in the network, we investigate their modulational instability (MI) and derive the analytical expression for the instability growth rate. We show that both the dissipative losses and the nonlinear dispersion constitute a great limiting factor to the experimental observation of the MI phenomenon. Investigating the transmission of modulated waves through the network, we show that the amplitude and the width of the solitonlike waves propagating in the network can be managed with the use of both the dissipative losses and the nonlinear dispersion of the network.

1. Introduction

The propagation of modulated waves such as bright solitons or dark solitons in various nonlinear physical systems, ranging from fluid dynamics to nonlinear optics through plasma physics and condensed matter physics, to nonlinear electrical transmission lines (NLTLs), has been the subject of considerable interest for many years [1–10]. On the other hand, nonlinear discrete electrical transmission lines are very convenient tools to study soliton propagation in 1D nonlinear dispersive media [3,5,8–16].

Nonlinear transmission lines provide a useful way to check how the nonlinear excitation behaves inside the nonlinear medium and to model the strange properties of new systems [4,15,17,18]. We think that these are the reasons why since pioneering works by Hirota and Suzuki [12] on NLTLs simulating Toda lattice, a growing interest has been devoted to the use of the NLTLs, in particular, for studying nonlinear waves and nonlinear modulated waves: pulse solitons, envelope pulse and hole solitons [7,8,13,19,20], discrete breathers [21], modulational instability [9,22,23]. Some years ago, Comte et al. used the NLTLs to experiment the propagation of compacton-like kinks in a diffusion–reaction chain [24]. Carrying theoretical studies on a Josephson-junction-loaded transmission line, Yaakobi et al. [25] recently shown that due to the nonlinearity of the system, a mixing process between four waves (wave-mixing) with different frequencies is possible.

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http://dx.doi.org/10.1016/j.cjph.2016.04.019

Most recently, Yemélé and Kenmogné introduced an extended nonlinear Schrödinger (NLS) equation to investigate the propagation of envelope dark solitary wave with compact support in a discrete nonlinear transmission line with nonlinear dispersion [26]. It is important to point out that in real physical systems, nonlinear dispersion can be introduced in order to improve the understanding of some physics phenomena, as for examples, the particles dispersion in suspension [28], the migration of magma [29], the formation of liquid drops and the dynamics of a dense anharmonic chain with many neighbors interaction or anharmonic lattices [30,31], the thermodynamic properties of anharmonic lattices [32], the phase dynamics of a chain of autonomous, self-sustained, dispersively coupled oscillators [33].

The purpose of this work is to investigate the dynamics of modulated waves in a dissipative version of Noguchi NLTL [27] with nonlinear dispersion. More precisely, we consider a NLTL which consists of a finite number of blocks connected as illustrated in Fig. 1. Each unit cell is modelled by a linear inductors L_1 shunted by a nonlinear capacitor $C_S(V)$ in the series branch and, in the shunted branch, a nonlinear capacitor $C(V)$ shunted by another linear inductance L_2. In order to take into account the dissipation of the network, a conductance G is connected in parallel with inductor L_2, accounting for the dissipation of L_2 in addition with the loss of the nonlinear capacitor C. Nonlinear element C is the well-known bias-dependent capacitor, responsible for nonlinearity of the system. The voltage-dependent capacitor C_S in the series branch induces a nonlinear dispersion in the network; it is introduced to allow the network to exhibit a variety of tasks, namely, the propagation of pulses, modulated waves, and compact envelope dark solitary waves. Therefore, the network of Fig. 1 exhibits the standard nonlinearity described by the bias-dependent capacitor $C(V)$, while the nonlinear capacitor $C_S(V)$ account for the nonlinear dispersion. By means of an extended nonlinear Schrödinger equation, Yemélé and Kenmogné proved the propagation of envelope dark solitary waves with compact support in the NLTL of Fig. 1 when the dissipative element was absent [26]. Some years ago, Gharakhili et al. [34] used the nonlinear Schrödinger equation to investigate soliton generation in left-handed nonlinear with series nonlinear capacitances and linear shunt inductances. Most recently, Wang et al. [35] experimentally investigated nonlinear properties of lattice network-based (LNB) composite right-/left-handed transmission lines (CRLH TLs) with nonlinear capacitors in series branches and show that harmonic generation, subharmonic generation, and parametric excitation are clearly observed in an unbalanced LNB CRLH TL separately. One decade ago, Kozyrevvan and Weide [36] have proven that nonlinear and anomalous dispersion of left-handed nonlinear transmission lines enables effective harmonic generation. Recently, Kengne et al. [7] have studied the dynamics of modulated in the dissipative NLTL of Fig. 1 in the case of a linear capacitor C_S. In this work, we aim to investigate the effects of the dispersion, and nonlinearity introduced respectively by the nonlinear capacitors C_S and C on modulated waves propagating through the network under consideration. The effects of the dissipative losses on wave propagation are also investigated. The rest of our work is organized as follows. In Section 2, we present the characteristics of the dissipative version of Noguchi model with nonlinear dispersion. In Section 3, by means of reductive perturbation method and the continuum limit approximation, we derive the envelope Ginzburg-Landau equation governing modulated waves in the network. The problem of the linear stability of Stokes waves is investigated in Section 4. After finding exact analytical solitonlike solutions for the obtained envelope GL equation in Section 5, we analytically investigate the propagation of solitonlike waves in the network. The main results of the paper are summarized in Section 6.

2. Characteristics of the dissipative version of Noguchi model with nonlinear dispersion

In the network of Fig. 1, the capacitance of the bias-dependent capacitor C is assumed to be expanded as a power series of the local signal voltage V_n, which appears across the nonlinear capacitor of the nth cell:

$$C(V_n + V_b) = \frac{dQ_n}{dV_n} = C_0\left(1 - 2\alpha V_n + 3\beta V_n^2\right), \tag{1}$$

where $C_0 = C_0(V_b)$ is a real constant corresponding to the capacitance of the nonlinear diode at the dc bias-voltage V_b, α and β are nonlinear parameters of the electrical stored charge Q_n and are assumed to be positive [26]; subscript n stands for the

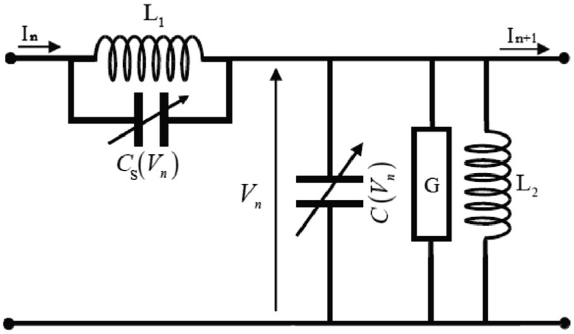

Fig. 1. Schematic representation of one unit cell of a dissipative version of Noguchi model with nonlinear dispersion. The network possesses N identical unit cells.