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Exponential rational function method for solving nonlinear equations arising in various physical models

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ABSTRACT

In this paper, we explore new applications of the exponential rational function method to the scalar Qiao equation and the Kuramoto-Sivashinsky equation. The obtained solutions may be of significance for the explanation of some practical physical problems. The method is very suitable, easy and effective handling of the solution process of nonlinear equations.

1. Introduction

In the recent past, scientists have seen a considerable interest in the investigation of nonlinear processes [1–3]. The reason for this is that they appear in various branches of natural sciences and particularly in almost all branches of physics: fluid dynamics, plasma physics, elastic media, field theory, nonlinear optics, control theory, systems identifications and condensed matter physics. With the development of soliton theory, various methods for obtaining the exact solutions of nonlinear partial differential equations (NPDEs) have been presented, such as the tanh function method [4], the extended tanh method [5], the Jacobi elliptic function method [6], the homogeneous balance method [7], the sine-cosine method [8], the F-expansion method [9], the exp-function method [10], the multiple exp-function method [11], the modified simple equation method [12,13], the transformed rational function method [14], the mapping method [15], the first integral method [16], the extended trial equation method [17,18], the (G'/G)-expansion method [19,20], the auxiliary equation method [21], Wronskian and Casoratian techniques [22] and so on [23–27].

In this paper, we aim to investigate new exact solutions of the scalar Qiao equation and the Kuramoto-Sivashinsky equation by using the exponential rational function method. In Section 2, we describe the mentioned method step by step. In Section 3, two applications of this method are given. Finally, in Section 4 some conclusions are stated.

2. Description of the exponential rational function method

We consider the general NPDE of the type:

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$$P(u, u_x, u_t, u_{xx}, u_{xt}, \ldots) = 0, \tag{1}$$

where $u = u(x,t)$ is an unknown function and P is a polynomial of u and its partial derivatives, in which the highest order derivatives and the nonlinear terms are involved. The main steps of this method can be given as follows [28]:

Step 1. Using the following transformation

$$u(x,t) = u(\xi), \xi = x + \omega t, \tag{2}$$

where ω is the speed of the wave, we reduce Eq. (1) to the following ordinary differential equation:

$$Q\left(u, u', u'', u''', \ldots\right) = 0. \tag{3}$$

Here the superscripts indicate the derivation with respect to ξ. According to possibility, Eq. (3) can be integrated term by term one or more times.

Step 2. Suppose that the exact solution of Eq. (3) can be expressed as follows:

$$u(\xi) = \sum_{n=0}^{N} \frac{a_n}{(1 + e^{\xi})^n}, \tag{4}$$

where a_n ($a_N \neq 0$) are constants to be determined later. Balancing the highest order linear term with the highest order nonlinear term in Eq. (3), we get the balancing number. N.

Step 3. Substituting Eq. (4) into Eq. (3) and collecting all terms with the same order of $e^{i\xi}$ ($i = 0,1,2,\ldots$) together, the left-hand side of Eq. (3) is transformed into another polynomial in $e^{i\xi}$. Equating each coefficient of this polynomial to zero yields a set of algebraic equations for a_n unknown parameters. Solving the equation system with the aid of Maple packet program, we can obtain the exact solutions of Eq. (1).

3. Applications

3.1. The scalar Qiao equation

Qiao et al. proposed a completely integrable equation [29]:

$$m_t = \frac{1}{2}\left(\frac{1}{m^k}\right)_{xxx} - \frac{1}{2}\left(\frac{1}{m^k}\right)_x. \tag{5}$$

which may have smooth solitons. When $k = 2$, this equation has been shown to have a bi-Hamiltonian structure and Lax pair that implies its integrability. Also it is shown in Ref. [30] that, for certain values of k (-2, $-½$, ½, 2) the equation may have infinitely many solitary wave solutions. When $k = ½$, Eq. (5)reads as a Harry Dym-type equation, which is actually the first member in the positive Camassa-Holm (CH) hierarchy [31].

To obtaining exact solutions of this equation we apply the travelling wave transformation (2) Therefore Eq. (5) takes the form:

$$\omega m = \frac{1}{2}\left(\frac{1}{m^k}\right)'' - \frac{1}{2}\left(\frac{1}{m^k}\right) = 0. \tag{6}$$

For solving this equation, we set

$$m = u^{\frac{1}{k+1}} \tag{7}$$

and Eq. (6) converted as