Chinese Journal of Physics 54 (2016) 69–76

Contents lists available at ScienceDirect

Chinese Journal of Physics

journal homepage: http://www.journals.elsevier.com/chinese-journal-of-physics/

Stochastic resonance in a fractional linear oscillator subject to random viscous damping and signal-modulated noise

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ARTICLE INFO

Article history:
Available online 24 March 2016

ABSTRACT

The stochastic resonance in a fractional linear system with random viscous damping driven by a dichotomous noise and signal-modulated noise is investigated. By the use of the properties of the noises and the Shapiro-Loginov formula and Laplace transform technology, the exact expression for the mean output-amplitude-gain (OAG) of the system is obtained. The non-monotonic influence of the viscous damping of the oscillator on the OAG is found. It is shown that the OAG is a non-monotonic function of the intensity and the correlation rate of the dichotomous noise. The OAG varies non-monotonically with the friction coefficient, with the frequency of the driving signal, with the frequency of the linear oscillator, as well as with the fractional exponent of the fractional oscillator.

1. Introduction

Stochastic forces can play a crucial role in influencing the deterministic kinetics. A representative example is stochastic resonance (SR) observed in systems driven by a combination of periodic and random forcing [1–7]. The original work on the SR by Benzi et al., was in the context of modeling the switch of the Earth's climate between ice ages and periods of relative warmth with a period of about 100 000 years [1,2]. Typically SR is a nonlinear effect that accounts for the optimum response of a dynamical system to an external forcing at a precise value of the noise intensity. The term SR in a broad sense means the non-monotonic behavior of the output signal as a function of some characteristics of the noise or of a periodic signal or of a parameter of the system studied.

A harmonic oscillator is very important model in physics, for example, any mass subject to a force in stable equilibrium acts as a harmonic oscillator for small vibrations [8–11]. Harmonic oscillators occur widely in nature and are exploited in many manmade devices, such as clocks and radio circuits. In classical mechanics, a harmonic oscillator is a system that, when displaced from its equilibrium position, experiences a restoring force, proportional to the displacement. In real oscillators, friction, or damping, slows the motion of the system. Due to frictional force, the velocity decreases in proportion to the acting frictional force. While simple harmonic motion oscillates with only the restoring force acting on the system, damped harmonic motion experiences friction. Mechanical examples include pendulums (with small angles of displacement), masses connected to springs, and acoustical systems. Electronic examples include electrical harmonic oscillators such as RLC (resistance-inductance-capacity) circuits.

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http://dx.doi.org/10.1016/j.cjph.2016.03.005
0577-9073/

Recently, the SR phenomenon for a fractional linear oscillator has been widely investigated. The resonance behavior for the output signal of a fractional oscillator with fluctuating frequency subjected to an external periodic force [12,13], to a periodically modulated noise [14], as well as to a signal-modulated noise [15], is considered. The resonant behavior of fractional harmonic oscillator with fluctuating mass is also investigated [16], which suggested that SR behavior can be induced by mass fluctuation noise.

Damping is an influence within or upon an oscillatory system that has the effect of reducing, restricting or preventing its oscillations. In physical systems, damping is produced by processes that dissipate the energy stored in the oscillation. Examples include viscous drag in mechanical systems, resistance in electronic oscillators, and absorption and scattering of light in optical oscillators. Damping not based on energy loss can be important in other oscillating systems such as those that occur in biological systems. Up to now, the previous works on SR for a fractional linear oscillator, the damping coefficient is mainly considered as a fractional friction [12–16]. However, a viscous damping does exist in an oscillator, such as in an integral-order linear oscillator [17–20] and in a Duffing oscillator [21]. As we know, little works have focused on SR effect for a fractional linear oscillator subjected both to a viscous damping and to a friction damping. Therefore, in this work, we investigated a fractional linear oscillator with random viscous damping and a fractional friction damping coefficient. The nonlinear dependence of the output of the fractional oscillator on the parameters of the noise and the system is found.

The structure of the work is as follows. In Section II, a fractional linear oscillator with random viscous damping and random frequency is introduced. Exact formulas are found for the output amplitude gain. In Section III, the SR phenomenon for the fractional oscillator is discussed. Finally some conclusions are drawn.

2. The fractional linear oscillator with random viscous damping and its output-amplitude-gain

Considering a fractional linear oscillator coupled with a noisy environment and a power-law-type memory friction kernel subjected random viscous damping and random frequency, described as the following stochastic differential equation

$$\frac{d^2x(t)}{dt^2}+[2r+\xi(t)]\frac{dx(t)}{dt}+\frac{k}{\Gamma(1-\alpha)}\int_0^t\frac{\dot{x}(u)}{(t-u)^{\alpha}}du+\left[\omega^2+\xi(t)\right]x(t)=\eta(t)A\cos(\Omega t), \tag{1}$$

where $x(t)$ is the oscillator displacement, r, k are the linear viscous damping and friction coefficient, respectively. α is the fractional exponent $(0<\alpha<1)$, $\Gamma(t)$ is the gamma function. In the present work, $\xi(t)$ fluctuates both the viscous damping and the frequency of the oscillator, which exists in the case of the fluctuation of the viscous damping and frequency being from the same source. For example, in an RLC circuit, the environment temperature and humidity can simultaneously affect the value of the inductor and capacitor. $\xi(t)$ is modeled as a Markovian dichotomous noise, which consists of jumps between two values a and $-a$, $a>0$, with stationary probabilities $p(a)=p(-a)=1/2$. Its mean and variance are

$$\langle\xi(t)\rangle=0,\langle\xi(t)\xi(t+\tau)\rangle=a^2e^{-\lambda|\tau|}=De^{-\lambda|\tau|}, \tag{2}$$

where λ is the correlation rate of the dichotomous noise, and D its intensity. $\eta(t)$ is a zero mean signal-modulated noise, with coupling strength P with noise $\xi(t)$, i.e., $\langle\xi(t)\eta(t)\rangle=P$.

Averaging Eq. (1), applying the characteristics of the noises $\xi(t)$ and $\eta(t)$, one gets

$$\frac{d^2\langle x\rangle}{dt^2}+2r\frac{d\langle x\rangle}{dt}+\left\langle\xi\frac{dx}{dt}\right\rangle+\frac{k}{\Gamma(1-\alpha)}\int_0^t\frac{\langle\dot{x}(u)\rangle}{(t-u)^{\alpha}}du+\omega^2\langle x\rangle+\langle\xi x\rangle=0. \tag{3}$$

Multiplicating both sides of Eq. (1) with $\xi(t)$, and then averaging, one gets

$$\left\langle\xi\frac{d^2x}{dt^2}\right\rangle+2r\left\langle\xi\frac{dx}{dt}\right\rangle+\left\langle\xi^2\frac{dx}{dt}\right\rangle+\frac{ke^{-\lambda t}}{\Gamma(1-\alpha)}\int_0^t\frac{\langle\xi(u)\dot{x}(u)\rangle e^{\lambda u}}{(t-u)^{\alpha}}du+\omega^2\langle\xi x\rangle+\langle\xi^2x\rangle=\langle\xi(t)\eta(t)\rangle A\cos(\Omega t), \tag{4}$$

Using the well-known Shapiro-Loginov procedure [22], one obtains

$$\frac{d\langle\xi x\rangle}{dt}=\left\langle\xi\frac{dx}{dt}\right\rangle-\lambda\langle\xi x\rangle, \tag{5}$$

Applying the characteristics of the fractional derivative and Shapiro-Loginov procedure, one can rewrite Eq. (4) as