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# Enhancement of single mode operation in coaxial optical waveguide using DB boundary conditions

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ABSTRACT

In this study, a competent numerical strategy to compute the dispersion of optical waveguides is presented and propagation of electromagnetic waves in a coaxial optical waveguide with DB boundary conditions is instigated. For this intend, cylindrical coordinates are here being used to derive the DB boundary conditions and to obtain field components for the modes. The propagation constant for the waveguide to be studied is determined by solving the Bessel and the modified Bessel functions. The cutoff frequencies for various lower order modes have been calculated and their dispersion characteristics are plotted correspondingly. The behavior of the coaxial optical waveguide under DB boundary conditions is shown to be significantly different from that of coaxial optical waveguide and conventional optical waveguide under traditional or tangential boundary conditions. Finally, the effect of waveguide dimensions on the mode cutoff frequencies and fabrication issues are also addressed.



## 1. Introduction

Dispersion is the major concern and one of the limiting factors that limits the available bandwidth in optical communication; as a result dispersion is a critical aspect at higher bit rate in a single mode waveguide [1]. Dispersion can be reduced by proper waveguide design. Generally, there are two approaches through which the dispersion characteristics of optical waveguide can be controlled first is by using smart medium in the waveguide and other is by smart structure of the waveguides [2]. In proposed study, authors have controlled the dispersion characteristics by the use of smart medium in which hypothetical interface exists at the boundary of the waveguide, where normal components of $D$ and $B$ vanish.

It is acknowledged that, electromagnetic wave propagations are usually defined by a set of differential equations and boundary conditions in a certain medium. When electromagnetic wave propagates from one medium to another medium the electromagnetic response of the medium changes, and it is mainly governed by boundary condition of the interface [3,4]. Electromagnetic boundary conditions are expressed traditionally in terms of tangential components of the fields at the boundary interface. For the wave to propagate or radiate in a certain medium, all the fields should be zero everywhere inside the perfect electric conductor (PEC) and perfect magnetic conductor (PMC) because tangential components of electric field ($E$) and magnetic field ($H$) across a metal is zero, which can be expressed in the following form [5,6]

$$\left.\begin{array}{l}\hat{n}\times\overline{E}=0\\ \hat{n}\times\overline{H}=\bar{J}_s\end{array}\right\}\quad \text{for PEC}$$

$$\left.\begin{array}{l}\hat{n}\times\overline{H}=0\\ \hat{n}\times\overline{E}=-M_s\end{array}\right\}\quad \text{for PMC} \tag{1}$$

where $\hat{n}$ is a vector normal to the surface of the given boundary interface, $\bar{J}_s$ and $M_s$ are the surface current density and magnetic displacement current density of the surface respectively. Kildal [7] has given a concept of artificially soft and hard surfaces boundary conditions for electromagnetic waves. According to him soft and hard surfaces boundary conditions are defined as

$$\left.\begin{array}{l}\hat{k}\cdot\overline{E}=0\\ \hat{k}\cdot\overline{H}=0\\ \hat{k}\cdot\hat{n}=0\end{array}\right\}\quad \text{for soft and hard surface} \tag{2}$$

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In the year of 2005, Lindell et al. [8,9] gave a novel concept of Perfect electromagnetic conductor (PEMC) for electromagnetic fields; it is defined using a scalar admittance parameter ($M$), whose zero and infinite limits yield the PMC and PEC surface respectively, which can be expressed as

$$\hat{n} \times (M\overline{E} + \overline{H}) = 0 \} \quad \text{for PEMC} \tag{3}$$

Boundary Eqs. (1)–(3) when defined in generalized form are known as is known as impedance boundary conditions and defined as [10]

$$\left.\begin{aligned} \hat{n} \times (\overline{E} - \overline{\overline{Z}} \cdot (\hat{n} \times \overline{H})) = 0 \\ \hat{n} \cdot \overline{\overline{Z}} = \overline{\overline{Z}} \cdot \hat{n} = 0 \end{aligned}\right\} \quad \text{Impedance conditions} \tag{4}$$

where $\overline{\overline{Z}}$ is the surface impedance dyadic, which may have infinite components.

In 2009, Lindell and Sihvola [11] introduced the set of boundary conditions in which required normal components of the $D$ and $B$ fields vanish at the boundary interface and are known as DB boundary conditions. These boundary conditions are mathematically defined as

$$\left.\begin{aligned} \hat{n} \cdot \overline{E} = 0 \\ \hat{n} \cdot \overline{H} = 0 \end{aligned}\right\} \tag{5}$$

The accomplishment of the DB boundary in terms of interface of uniaxially anisotropic medium with dyadic permittivity and permeability along with one parameter transverse and one parameter normal to the interface. The DB conditions are obtained from the continuity conditions by making normal permittivity and permeability parameters become zero [12,13]. Zero-valued medium parameters have been recently under great intrigue and their realization with alternating regions of opposite-valued permittivity and/or permeability has been one of the fields of research in metamaterials [14–17]. Practically, it is yet to be found the realization of DB boundary conditions in physical medium.

The behaviors of the modes of a coaxial waveguide are shown in Fig. 1. Using DB boundary conditions [11,18] the modal characteristic equation is derived with the help of Bessel function for the coaxial waveguide. Computed results are discussed in terms of dispersion curves and the dimension of the proposed waveguide. The results are compared with dispersion curves of the standard coaxial waveguide under tangential boundary conditions and also with the weakly –guiding step index fiber [1].

The paper is organized as follows; in Section 2 the characteristic equations are derived from Helmholtz's wave equation and then the characteristics of dispersion relation, cutoff frequencies are calculated with the help of Bessel functions using DB boundary conditions. In Section 3 numerical results and discussion are considered. Finally, the paper ends with some remarks in the conclusion given in Section 4.

## 2. Formulation of the problem

The problem of concern is the wave propagation properties of the normal modes of structure as shown in Fig. 1. This structure consists of dielectric medium with inner and outer interface bounded by DB boundaries. In order to investigate the problem, it is convenient to work in a coordinate system in which the boundary of the structure coincides with one of the coordinates which is being held constant. It is thus appropriate for the geometry under consideration to work in an orthogonal cylindrical coordinate system described by the coordinates $r$, $\phi$ and $z$.

The DB boundary conditions are located at $r = r_1$ and $r = r_2$ in the proposed waveguide. In order to obtain the electric ($E$) and magnetic ($H$) fields, the vector Helmholtz wave equation must be solved which is obtained from Maxwell's equations. The fields are assumed to vary harmonically in time and the wave propagates in the positive $z$ direction. After some mathematical steps, the scalar-wave function can be represented as the product:

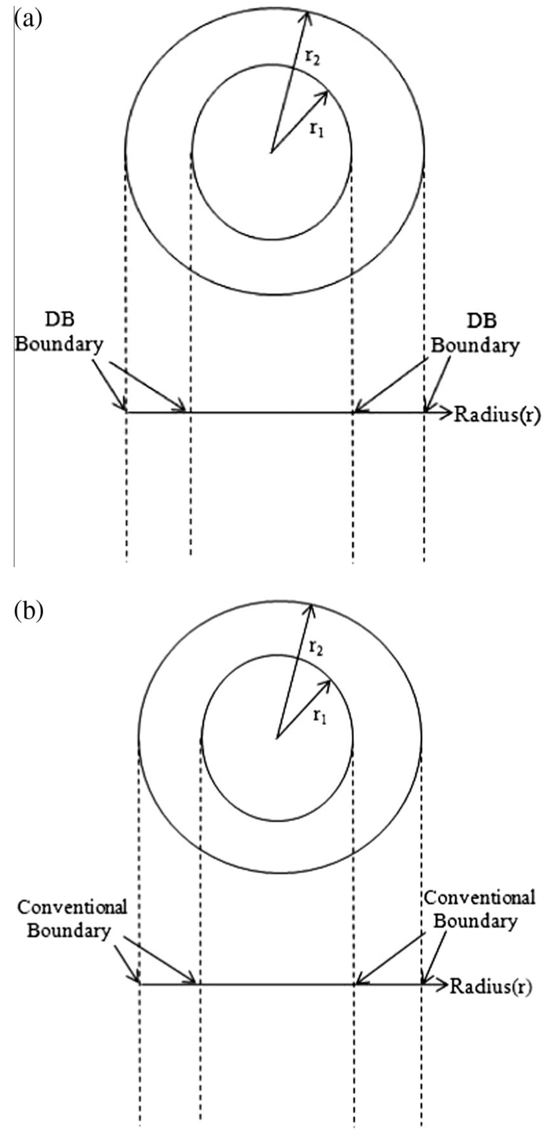


**Fig. 1.** The cross sectional view of the coaxial optical waveguide using (a) conventional boundary conditions and (b) DB boundary conditions.

$$\psi(r, \phi, z, t) = e^{-i\omega t} e^{i\beta z} \xi(r, \phi) \tag{6}$$

where $\psi$ represents either the $E_z$ or $H_z$ field amplitudes, $\xi(r, \phi)$ is the radial function describing the field amplitude in the $\xi$–$\eta$ plane and $\beta$ is the longitudinal component of the propagation constant. The physically acceptable solutions for annular sections for the proposed waveguide are:

$$\left.\begin{aligned} E_z = (AJ_v(qr) + BY_v(qr)) \\ H_z = (CJ_v(qr) + DY_v(qr)) \end{aligned}\right\} \tag{7}$$

where $q = \sqrt{(n_1^2 k_0^2 - \beta^2)}$ corresponding to refractive index ($n_1$) of dielectric medium between inner and outer interface bounded by DB boundaries, $k_0 = \frac{2\pi}{\lambda}$, is a wave number in free space and $\lambda$ is the wavelength of light in free space. $J_v$ and $Y_v$ are the Bessel functions of first kind and second kind respectively and $A$, $B$, $C$, $D$, all are unknown constants. Now, equations of $E_\phi$ and $H_\phi$ are defined in terms of Bessel functions for the proposed waveguide as