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Generalized thermal radiation from arbitrary fractional dimension

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HIGHLIGHTS

- Thermal radiation properties of fractional dimension are investigated.
- Energy density is calculated as a function of arbitrary fractional dimension.
- Maximum frequency factor is shown as a function of fractional dimension.
- Measurement method of fractional dimension is shown.

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ABSTRACT

Properties of the thermal radiation from arbitrary fractional dimension are investigated. Generalized blackbody radiation for arbitrary dimension can be obtained and the energy density is shown as a function of arbitrary dimension as well as temperature. Maximum frequency factor representing the relation between most probable photon energy and thermal energy is shown as a function of arbitrary fractional dimension. It is also shown how to measure the arbitrary fractional dimension of the body with thermal radiation.

1. Introduction

Blackbody radiation has been widely used to measure the temperature of a body. Stefan–Boltzmann's law for integer dimension was investigated [1]. Thermal radiation becomes very different from blackbody radiation when the size of the body is reduced. The size effect of thermal radiation was investigated in one, two, and three dimensions [2–4]. Non-uniformity correction techniques for infrared detector and camera to measure temperature accurately were developed [5–8] and minimum resolvable temperature was predicted in thermal imaging sensing [9]. Effective temperature was calculated for non-uniform temperature distribution [10–12]. Fractal dimension has been investigated well in many different research fields such as physics, mathematics, and chaos [13–15]. In reality, integer dimension is an ideal one, but nature shows fractional dimension. These fractional dimensions become important especially in low dimensional nanostructures. For instance, nanowire having curvature can show fractional dimension which is higher than one-dimension and thin film having surface roughness can show fractional dimension which is higher than two-dimension. Based on fractal characterization, the role of the rough surface structure on the thermal and hydrodynamic properties in micro-channels was evaluated using a computational fluid dynamic simulation [16]. Heat capacity of liquid helium was calculated in a fractal dimension between two and three dimension, and those changes were large in spite of the small variation of the fractal dimension [17]. Therefore, it is important to measure the fractional dimension of the body in order to understand physics properties. There has not done any connection between fractional dimension and thermal radiation.

In this paper, we investigate the properties of thermal radiation from arbitrary fractional dimensions. The energy density of thermal energy is increased as fractional dimension increases. Maximum frequency factor which shows the relation between most probable photon energy and thermal energy is shown as a function of arbitrary fractional dimension.

2. Fractional dimension

We are familiar with integer space dimensions such as one, two and three while time is always one-dimension. However, nature also shows fractional dimensions. These fractional dimensions are important in low dimensional nanostructure materials, which

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are below three-dimension. For instance, a bended wire shows fractional dimension which is higher than one-dimension and flat surface having surface roughness shows fractional dimension which is higher than two-dimension.

Fractal dimension for Koch curve can be calculated as

$$D = \frac{\ln N}{\ln\left(\frac{1}{S}\right)}, \tag{1}$$

where D is the dimension, N is the number of pieces, and S is the scaling factor. Fractals have a fractional dimension which is not integer dimension and are self-similar in which a small portion of a fractal looks similar to whole object. For $S = 1/3$, one-dimension shows $N = 3$, two-dimension shows $N = 9$, and three-dimension shows $N = 27$. The scaling factor can be any number to determine arbitrary dimension. From Eq. (1), we can find arbitrary fractional dimension for any given geometry.

3. Thermal radiation from integer dimension

Thermal radiation from integer dimension is well-known. For one-dimensional blackbody radiation, the energy density is [3]

$$u_B(T) = \left(\frac{2\pi^2}{3}\right)\left[\frac{(k_BT)^2}{(hc)}\right], \tag{2}$$

where h is the Planck constant, c is the speed of light and k_B is the Boltzmann constant. For two-dimensional blackbody radiation, the energy density becomes [4]

$$u_B(T) = 8\pi\zeta[3]\left[\frac{(k_BT)^3}{(hc)^2}\right], \tag{3}$$

where ζ is the Riemann zeta function. For three-dimensional blackbody radiation, the energy density is [2]

$$u_B(T) = \left(\frac{8\pi^5}{15}\right)\left[\frac{(k_BT)^4}{(hc)^3}\right]. \tag{4}$$

From Eqs. (2)–(4), the energy density of integer space dimension is directly proportional to the total dimensional power of the temperature whose total dimension includes space dimension and time dimension. Space can be any dimension, but time is always one-dimension.

4. Thermal radiation from arbitrary fractional dimension

The surface area for arbitrary dimension in Euclidean space is $S_D\,(R) = \frac{2\pi^{(D+1)/2}}{\Gamma[(D+1)/2]}R^D$ and the volume is $V_D(R) = \frac{\pi^{D/2}}{\Gamma\left(\frac{D}{2}+1\right)}R^D$ where D represents the dimension and R represents the radius of Euclidean plane. The surface area and volume are increased as the dimension increases.

Energy density of blackbody radiation for arbitrary dimension can be generalized as [1]

$$u_B(D,T) = \frac{4\pi^{D/2}\Gamma(D+1)\zeta(D+1)}{\Gamma(D/2)}\left(\frac{(k_BT)^{D+1}}{(hc)^D}\right), \tag{5}$$

where Γ is the gamma function, ζ is the Riemann zeta function and D is the space dimension. Energy density is proportional to the $(D+1)$ th power of temperature in which D represents space dimension and one represents time dimension. From Eq. (5), the energy density can be calculated for arbitrary dimension and temperature. For $D = 1$, 2, and 3 in Eq. (5), we can get one-dimensional blackbody radiation of Eq. (2), two-dimensional blackbody radiation of Eq. (3), and three-dimensional blackbody radiation of Eq. (4), respectively. Fig. 1 shows the energy density as a function of dimension and temperature. The energy density is increased as dimension increases. We can notice that energy density is a continuous function of dimension. The energy density is also increased as temperature increases.

The spectral radiance for arbitrary dimension can be expressed as

$$\Phi_\nu = \frac{4\pi^D R^D}{(\Gamma[(D+1)/2])^2}\frac{(k_BT)^D}{(hc)^{D-1}}\frac{(h\nu/k_BT)^D}{\exp(h\nu/k_BT)-1}, \tag{6}$$

where R is the radius of Euclidean plane. Fig. 2 shows the spectral radiance as a function of dimensionless photon energy $\left(\frac{h\nu}{k_BT}\right)$ for different dimensions. Spectral radiance is increased as dimension increases for constant temperature. Spectral radiance depends strongly on the fractional dimension. Fig. 3 shows the spectral radiance as a function of dimensionless photon energy $\left(\frac{h\nu}{k_BT}\right)$ for different temperatures in two-dimension. Spectral radiance is also increased as temperature increases for constant dimension.

Maximum frequency which is the most probable photon frequency can be found from the derivative of Eq. (6)

$$\frac{Dx^{D-1}}{e^x-1} - \frac{x^De^x}{(e^x-1)^2} = 0, \tag{7}$$

where $x = \frac{h\nu}{k_BT}$ represents the dimensionless photon energy in which the photon energy is divided by thermal energy and D is the arbitrary dimension of the body. From Eq. (7), the maximum frequency factor which shows the relation between the most probable photon energy and thermal energy can be numerically found as

$$\alpha(D) = \frac{h\nu_{\max}}{k_BT}, \tag{8}$$

where α depends on dimension. Eq. (8) represents generalized Wien's displacement law. Table 1 shows the maximum frequency factor vs. arbitrary fractional dimension. From Table 1, we can find that the maximum frequency factor for three-dimension, $\alpha = 2.82144$, which is well-known. Fig. 4 shows the maximum frequency factor as a function of arbitrary fractional dimension. Maximum frequency is related to dimension as well as temperature. Maximum frequency factor is increased as dimension increases. The maximum frequency factor is increased almost linearly with the dimension when the dimension increases above 4. The maximum frequency factor for high dimension can be useful in string theory.

Arbitrary fractional dimension can also be understood by considering the number of photon of the body. The number of photon is proportional to the Dth power of kL, $(kL)^D$, where k is the photon wave number, D is the arbitrary dimension and L is the length of the body. The number of photon is proportional to $(kL)^1$ for one-dimension, $(kL)^2$ for two-dimension and $(kL)^3$ for three-dimension.

The dimension of a body can be found by measuring the maximum frequency factor as shown in Eq. (8) or Table 1. For a constant temperature, the maximum frequency factor is increased as the dimension of the body increases. For instance, the maximum frequency is increased as the surface roughness increases. It is expected that we can measure the fractional dimension of the body by measuring the maximum frequency factor from thermal radiation.

A rock's fracture surface was determined with the use of the triangular prism surface area and projective covering methods [18]. There are many different methods to measure fractional dimension and its value depends on how to measure. Our method can be the most universal method to measure fractional dimension with thermal radiation, which can be applied to artificial as well as natural surface.