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# Principal curvature for infrared small target detection

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## HIGHLIGHTS

- Principal curvature is introduced for infrared small target detection.
- The filter function is the product of positive Gaussian curvature and negative mean curvature.
- An approximate model is provided for optimizing the parameters and verifying the effectiveness of the method.
- The proposed method outperforms other popular methods.

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## ABSTRACT

Small target detection in infrared image with complex background and low signal–noise ratio is an important and difficult task in the infrared target tracking system. In this paper, a principal curvature-based method is proposed. The principal curvatures of target pixels are negative and their absolute values are larger than that of background pixels and noise pixels in a Gaussian-blurred infrared image. The proposed filter takes a composite function of the curvatures for detection. An approximate model is also built for optimizing the parameters. Experimental results show that the proposed algorithm is effective and adaptable for infrared small target detection in complex background. Compared with several popular methods, the proposed algorithm demonstrates significant improvement on detection performance in terms of the parameters of signal clutter ratio gain, background suppression factor and ROC.



## 1. Introduction

The detection of small target is an important subject in Infrared Search and Track (IRST) surveillance systems, which apply to many areas such as infrared imaging guidance systems, space surveillance systems, astronomy prognosticates, remote sensing and forest warning and so on [1]. The target is fairly small and the SCR (signal-clutter ratio) is fairly low when the detection distance reaches 20 km or longer [2]. In recent years, many research works dedicated to the infrared small target detection have emerged [3–13]. In general, the detection algorithms include two parts: background suppression and detection/tracking (tracking before detection or detection before tracking). Conventional background suppression methods are mainly filter-based methods, such as morphological filter [3–5], 2D least mean squares filter [6,7], low pass filters [8], wavelet filter [9] etc. These methods are usually low time-consumption, and perform well when SCR is high. However, these methods would result in serious false alarms and degraded detection performance when the SCR is low. Therefore, the key problem is how to improve the performance of filter-based methods on the occasion of low SCR environment. In recent years, researches related to statistical learning methods have been done, such as evidence combination approach [10], principal component analysis [11], linear discriminant vector [12], kernel fuzzy C-means [13]. In general, statistical learning methods are adaptive for varied application environment. However the drawbacks of these methods cannot be ignored. For example, the training processes are hard to perform, usually a large-size training dataset should be collected and labeled at first, and the target types (varied sizes, brightness, shapes, etc.) in dataset should be as many as possible for robust detection. In addition, the detection performances of the statistical learning methods are usually lower than that of well-designed filter-based methods. The above drawbacks narrow the usage of the statistical learning methods.

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In this paper, we propose a new infrared small target detection method based on principal curvature. The method is derived by the geometry meaning of curvature of the image surface. In this method, the image is initially smoothed by a Gaussian filter. The standard deviation of the Gaussian filter has a significant influence on the detection result and should be selected deliberately. Then the principal curvatures of each pixel are calculated by the eigenvalues of Hessian matrix, and the Gaussian curvature and mean curvature, which are the product value and mean value of the principal curvature respectively, are calculated as well. Targets can be enhanced and clutters can be suppressed significantly in the images of Gaussian curvature and mean curvature. At last, the product of the positive Gaussian curvature and negative mean curvature are calculated for detection. Experiments demonstrate that the proposed method shows better detection performance compared with several widely used methods.

## 2. Principal curvature

In differential geometry, the curvature at a given point of a surface measure how the surface bends by different amounts at different directions. The principal curvatures are the maximum and minimum values of this curvature. Additionally, the directions where the curvature takes its maximum and minimum values are always perpendicular, and are called principal directions. Principal curvature and principal direction have been applied for curvilinear structures detection [14] edges and ridges detection [15–17], and object recognition [18] et al. They show good performance in the applications such as road detection and blood vessel detection. In this paper, principal curvature is introduced for infrared small target detection.

The principal curvatures are calculated by Hessian matrix approximately, which is a symmetric square matrix of second order partial derivatives of a function. An image can be regarded as a two-dimensional function which is given as

$$z = f(x, y) \tag{1}$$

Thus the form of the Hessian matrix is given as

$$\mathbf{H} = \begin{bmatrix} f_{xx} & f_{xy} \\ f_{yx} & f_{yy} \end{bmatrix} \tag{2}$$

Since the surface of an image is discrete, the derivatives are calculated approximately by finite central differences. Two key concepts related to the principal curvatures are Gaussian curvature and mean curvature. In detail, Gaussian curvature is the product of the two principal curvatures and mean curvature is the mean of the two principal curvatures. Gaussian curvature $K$ and mean curvature $H$ are calculated as

$$\begin{aligned} K &= \frac{f_{xx}f_{yy} - f_{xy}^2}{(1+f_x^2+f_y^2)^2} \\ H &= \frac{(1+f_x^2)f_{yy} - 2f_xf_yf_{xy} + (1+f_y^2)f_{xx}}{2(1+f_x^2+f_y^2)^{3/2}} \end{aligned} \tag{3}$$

When the gradient is equal to 0 (i.e. $f_x = 0$ and $f_y = 0$), Gaussian curvature and mean curvature are calculated as

$$\begin{aligned} K &= f_{xx}f_{yy} - f_{xy}^2 = |\mathbf{H}| \\ H &= \frac{f_{yy} + f_{xx}}{2} = \frac{tr(\mathbf{H})}{2} \end{aligned} \tag{4}$$

where $\mathrm{tr}(\cdot)$ donates the trace of a matrix and $|\cdot|$ donates the determinant of a matrix. It is indicated by formula (4) that principal curvatures of the surface at point $p$ are the eigenvalues of the Hessian matrix when the point p is a critical point (i.e. the gradient of the point p vanishes). When the gradient is not 0, the principal curvatures cannot be calculated by Hessian matrix accurately, nevertheless, taking the approximate form will usually yield good detection performance. However, taking the accurate form in formula (3) to calculate principal curvatures will usually result in degraded detection performance, for this reason, we use the approximate principal curvatures for detection.

## 3. Proposed method

### 3.1. Gaussian blurring

Infrared image contains targets, backgrounds and noises. The curvature near the noise is a large value, which will interfere with the detection of targets. In order to smooth the image and reduce the influence of the noises, a Gaussian filter is applied in the detection process. The two-dimensional Gaussian filter is given as:

$$g(u, v, \sigma) = \frac{1}{2\pi\sigma^2} e^{-(u^2+v^2)/(2\sigma^2)} \tag{5}$$

where $\sigma$ is the standard deviation of the Gaussian distribution.

As shown above, the Gaussian filter is decided by its standard deviation. On one hand, a small standard deviation may not reduce the influence of the noises, on the other hand, a large standard deviation may dim the target and make it difficult to detect target. Therefore the standard deviation of the filter should be selected deliberately.

### 3.2. Detection function

Target is usually brighter than nearby pixels in infrared image. Therefore, the surface near the target is a concave surface. As a result, the two principal curvatures are negative, i.e., the two eigenvalues of the Hessian matrix near the target are both negative. The proposed detection function is given as

$$f_d = \max(e_1e_2, 0) \cdot \max(-e_1 - e_2, 0) \tag{6}$$

where $e_1$ and $e_2$ are eigenvalues of Hessian matrix. Product and sum of the eigenvalues are calculated as:

$$\begin{aligned} e_1 + e_2 &= tr(\mathbf{H}) = f_{xx} + f_{yy} \\ e_1e_2 &= |\mathbf{H}| = f_{xx}f_{yy} - f_{xy}^2 \end{aligned} \tag{7}$$

Formula (7) indicates that the filter function is based on Gaussian curvature and mean curvature. After filtering, a global threshold is chosen to separate the targets candidates from backgrounds. The detection criterion can be given as the following:

$$f_d = \begin{cases} \text{Target pixel}, f_d(m,n) \geqslant Th \text{ and } f_d(m,n) \text{ is local maximum} \\ \text{Non-target pixel, otherwise} \end{cases} \tag{8}$$

where $Th$ is the segmenting threshold. The threshold is related to the size of target, infrared imaging device and weather and so on. It can be assigned by experience or the criterion which is introduced in the following part.

### 3.3. Parameter optimization

The standard deviation of the Gaussian filter and the segmenting threshold have a significant influence on the detection result. This part is focused on selecting a proper standard deviation and a segmenting threshold on the base of an approximate model. Infrared image consists of target, noise and background. The model of infrared image can be described as:

$$f(x, y) = f_T(x, y) + f_N(x, y) + f_B(x, y) \tag{9}$$

where $f(x,y)$, $f_T(x, y)$, $f_N(x, y)$ and $f_B(x, y)$ donate the intensity of infrared image, target, background and noise at point $(x, y)$