Infrared Physics & Technology 65 (2014) 5–8

Contents lists available at ScienceDirect

Infrared Physics & Technology

journal homepage: www.elsevier.com/locate/infrared

# On the relationship between direct and anisotropic diffuse radiation

Feng Zhang [a,b,c,*], Xiuji Zhou [c], Hua Zhang [d], Xindong Peng [c], Zhili Wang [e]

[a] Key Laboratory of Meteorological Disaster of Ministry of Education, Nanjing University of Information Science and Technology, Nanjing 210044, China
[b] University of Chinese Academy of Sciences, Beijing 100049, China
[c] State Key Laboratory of Severe Weather, Chinese Academy of Meteorological Sciences, Beijing 100081, China
[d] Laboratory for Climate Studies, National Climate Center, China Meteorological Administration, Beijing 100081, China
[e] Chinese Academy of Meteorological Sciences, Beijing 100081, China

## HIGHLIGHTS

- Relationship between the direct and anisotropic diffuse radiation.
- A summation of a series of the solution for direct radiation.
- 2N-stream discrete ordinates method.

## ARTICLE INFO

*Article history:*
Received 16 December 2013
Available online 1 March 2014

*Keywords:*
Radiative transfer
Direct radiation
Anisotropic diffuse radiation
Discrete ordinates method

## ABSTRACT

The relationship between the direct and anisotropic diffuse radiation for discrete ordinates method (DOM) is analyzed. The solution of anisotropic diffuse radiation can be expressed as a summation of a series of the solution for direct radiation in multi-stream DOM. A wide range of accuracy checks for anisotropic diffuse radiation of a homogeneous layer is performed with respect to the "exact" results computed from the $\delta$-128-stream scheme for radiative transfer. In addition, the results indicate that the increasing number of streams produces a decrease in error of reflection/transmission.



## 1. Introduction

Radiative transfer process is a key issue for remote sensing and climate modeling. There are two types of radiative transfer, and the corresponding results in reflection/transmission are treated separately as the direct and diffuse. The direct radiation is associated with the direct incoming beam and the diffuse radiation is associated with the incoming radiation. The solutions of radiative transfer for the direct and diffuse radiation are obtained in different ways. In general, the diffuse radiation involves the angular integral, which is more complex than the direct radiation. For infrared radiation, a lot of effort has been devoted to find an approximate approach to deal with the relationship between the direct and diffuse radiation [1–9]. Elsasser [1] first suggested that diffuse radiation is equivalent to direct radiation with an averaged cosine of the local zenith 1/1.66, which is the well known diffusivity factor method; Li [6] showed the diffusivity factor method is equivalent to the one-node Gaussian quadrature $\sqrt{e} = 1.6487$, which has a more solid mathematical basis; Shi and Qu [5] proposed a formula to represent the diffusivity factor as a function of optical depth; Zhang and Shi [7] proposed an improved formula for the diffusivity factor; Zhao and Shi [9] reinvestigated the problem of diffusivity factor by using the bridging function method. For the solar radiation, the diffuse radiation is also very important for radiative transfer process in the atmosphere [10–12]. Recently, the relationship between direct and diffuse radiation was showed by [13,14], which helps us to understand the physical essentials of the diffuse radiation for DOM. However, the results of [13,14] justly focused on the isotropic diffuse radiation, which is a special case of anisotropic diffuse radiation. In this work, we will show the relationship between direct and anisotropic diffuse radiation for the $\delta$-2N-stream DOM. For the isotropic diffuse radiation, the accuracy of reflection/transmission has been discussed in [13,15] for the $\delta$-two-stream approximation (D2S), $\delta$-four-stream approximation (D4S), $\delta$-six-stream approximation (D6S), and $\delta$-eight-stream approximation (D8S). In the following, we will show the result of DOM for the anisotropic diffuse radiation.

* Corresponding author at: Key Laboratory of Meteorological Disaster of Ministry of Education, Nanjing University of Information Science and Technology, Nanjing 210044, China. Tel.: +86 18918206092.

*E-mail address:* feng_zhang126@126.com (F. Zhang).

http://dx.doi.org/10.1016/j.infrared.2014.02.002
1350-4495/

## 2. Relationship between the direct and anisotropic diffuse radiation

The azimuthally averaged radiative transfer equation for the solar intensity $I(\tau,\mu)$ in a plane-parallel atmosphere is,

$$\mu\frac{dI(\tau,\mu)}{d\tau} = I(\tau,\mu) - \frac{\omega}{2}\int_{-1}^{1} I(\tau,\mu')P(\mu,\mu')\,d\mu', \tag{1}$$

where $\mu$ is the cosine of the local zenith angle, positive for downward and negative for upward directions; $\tau$ is the optical depth; $\omega$ is the single scattering albedo; $P(\mu,\mu')$ is the azimuthally-averaged scattering phase function defining the light incidence at $\mu'$ and scattered away at $\mu$. The upward and downward fluxes are

$$F^{\pm}(\tau) = 2\pi\int_0^1 I(\tau,\pm\mu)\mu\,d\mu.$$

### 2.1. Direct radiation

For direct radiation, there is a direct solar beam hitting the upper level of the considered layer and there is no upward diffuse radiance at the bottom of the layer.

$$\mu\frac{dI(\tau,\mu)}{d\tau} = I(\tau,\mu) - \frac{\omega}{2}\int_{-1}^{1} I(\tau,\mu')P(\mu,\mu')\,d\mu', \tag{2a}$$

$$I(0,-\mu) = \delta(\mu,\mu_0)\frac{F_0}{2\pi}, \tag{2b}$$

$$I(\tau_0,\mu) = 0, \tag{2c}$$

where $\tau_0$ is the total optical depth of the layer; $\mu_0$ is the cosine of the solar zenith angle, which is positive. The radiation field can be decomposed into two parts ([13,14]):

$$I(\tau,\mu) = I_{dir}(\tau,\mu) + I_{dif}(\tau,\mu),$$

where $I_{dir}(\tau,\mu)$ is the direct solar beam (photons without experiencing scattering), $I_{dif}(\tau,\mu)$ is the diffuse beam (photons experiencing at least one event of scattering). The direct solar beam satisfies $I_{dir}(\tau,\mu) = 0\ (\mu \geqslant 0)$, and $I_{dir}(\tau,-\mu) = \delta(\mu,\mu_0)\frac{F_0}{2\pi}e^{-\tau/\mu_0}\ (\mu \geqslant 0)$, respectively [13,14]. In addition, the diffuse beam $I_{dif}(\tau,\mu)$ satisfies [13,14]

$$\mu\frac{dI_{dif}(\tau,\mu)}{d\tau} = I_{dif}(\tau,\mu) - \frac{\omega}{2}\int_{-1}^{1} I_{dif}(\tau,\mu')P(\mu,\mu')\,d\mu' - \frac{\omega F_0}{4\pi}e^{-\tau/\mu_0}P(\mu,-\mu_0). \tag{3a}$$

$$I_{dif}(0,-\mu) = 0, \tag{3b}$$

$$I_{dif}(\tau_0,\mu) = 0. \tag{3c}$$

Eq. (3) is the well-known radiative transfer equation for diffuse beam of direct radiation [16]. The relationship between direct and isotropic diffuse radiation for two-stream and multi-stream DOM is discussed by [13,14]. In the follow, we will discuss the relationship between direct and anisotropic diffuse radiation.

### 2.2. Anisotropic diffuse radiation

For anisotropic diffuse radiation, the incoming radiation is expressed by $A(\mu)$. Therefore, at the upper level of a considered layer, $I(0,-\mu) = A(\mu)\quad(\mu \geqslant 0)$. Also there is no upward diffused flux at the bottom of the layer. The radiative transfer equation is

$$\mu\frac{dI(\tau,\mu)}{d\tau} = I(\tau,\mu) - \frac{\omega}{2}\int_{-1}^{1} I(\tau,\mu')P(\mu,\mu')\,d\mu', \tag{4a}$$

$$I(0,-\mu) = A(\mu), \tag{4b}$$

$$I(\tau_0,\mu) = 0, \tag{4c}$$

The exact solution of (4) is also difficult to obtain, the approximate approaches of DOM or spherical harmonic expansion method (SHM) have to be used. In the follows, we will present a solution in terms of the physical nature of DOM. If $A(\mu)$ is equal to constant, (4) is used to describe the process of isotropic diffuse radiation.

In (4b), by using the N-node Gaussian quadrature we have

$$2\pi\left[\int_0^1 I(\tau,-\mu)\mu\,d\mu\right]\Big|_{\tau=0} = 2\pi\sum_{i=1}^{N} a_i\mu_i I(\tau,-\mu_i) \tag{5}$$

where $\mu_i$ is obtained in [17]. Based on (5), the boundary condition (4b) becomes

$$I(0,-\mu) = \sum_{i=1}^{N} a_i A(\mu_i)\delta(\mu,\mu_i). \tag{6}$$

Thus, (4) yields

$$\mu\frac{dI(\tau,\mu)}{d\tau} = I(\tau,\mu) - \frac{\omega}{2}\int_{-1}^{1} I(\tau',\mu)P(\mu,\mu')\,d\mu', \tag{7a}$$

$$I(0,-\mu) = \sum_{i=1}^{N} a_i A(\mu_i)\delta(\mu,\mu_i), \tag{7b}$$

$$I(\tau_0,\mu) = 0. \tag{7c}$$

We decompose the radiation field into $N$ parts: $I(\tau,\mu) = I_1(\tau,\mu) + \cdots + I_{N-1}(\tau,\mu) + I_N(\tau,\mu)$, where $I_1(\tau,\mu),\ldots,I_{N-1}(\tau,\mu)$,and $I_N(\tau,\mu)$ satisfy

$$\mu\frac{dI_1(\tau,\mu)}{d\tau} = I_1(\tau,\mu) - \frac{\omega}{2}\int_{-1}^{1} I_1(\tau',\mu)P(\mu,\mu')\,d\mu', \tag{8a}$$

$$I_1(0,-\mu) = \delta(\mu,\mu_1)a_1A(\mu_1), \tag{8b}$$

$$I_1(\tau_0,\mu) = 0, \tag{8c}$$

$\ldots,$

$$\mu\frac{dI_{N-1}(\tau,\mu)}{d\tau} = I_{N-1}(\tau,\mu) - \frac{\omega}{2}\int_{-1}^{1} I_{N-1}(\tau',\mu)P(\mu,\mu')\,d\mu', \tag{9a}$$

$$I_{N-1}(0,-\mu) = \delta(\mu,\mu_{N-1})a_{N-1}A(\mu_{N-1}), \tag{9b}$$

$$I_{N-1}(\tau_0,\mu) = 0. \tag{9c}$$

and,

$$\mu\frac{dI_N(\tau,\mu)}{d\tau} = I_N(\tau,\mu) - \frac{\omega}{2}\int_{-1}^{1} I_N(\tau',\mu)P(\mu,\mu')\,d\mu', \tag{10a}$$

$$I_N(0,-\mu) = \delta(\mu,\mu_N)a_NA(\mu_N), \tag{10b}$$

$$I_N(\tau_0,\mu) = 0. \tag{10c}$$

The solutions of the diffuse radiation can be obtained from that of the direct radiation by replacing $2a_1A(\mu_1) \to F_0/\pi,\ \mu_1 \to \mu_0,\ldots,$ $2a_{N-1}A(\mu_{N-1}) \to F_0/\pi,\ \mu_{N-1} \to \mu_0$ and $2a_NA(\mu_N) \to F_0/\pi, \mu_N \to \mu_0$ in (8)–(10), respectively. Therefore, we obtain

$$\bar{r} = \frac{[a_1\mu_1A(\mu_1)r(\mu_1) + \cdots + a_{N-1}\mu_{N-1}A(\mu_{N-1})r(\mu_{N-1}) + a_N\mu_NA(\mu_N)r(\mu_N)]}{a_1\mu_1A(\mu_1) + \cdots + a_{N-1}\mu_{N-1}A(\mu_{N-1}) + a_N\mu_NA(\mu_N)}, \tag{11a}$$

$$\bar{t} = \frac{[a_1\mu_1A(\mu_1)t(\mu_1) + \cdots + a_{N-1}\mu_{N-1}A(\mu_{N-1})t(\mu_{N-1}) + a_N\mu_NA(\mu_N)t(\mu_N)]}{a_1\mu_1A(\mu_1) + \cdots + a_{N-1}\mu_{N-1}A(\mu_{N-1}) + a_N\mu_NA(\mu_N)}. \tag{11b}$$

where $r(\mu_1),\ldots,r(\mu_{N-1})$ and $r(\mu_N)$ are the results of $r(\mu_0)$ in (2) by replacing $\mu_0 \to \mu_1,\ldots,\mu_0 \to \mu_{N-1}$ and $\mu_0 \to \mu_N$, respectively. The same is for $t(\mu_1),\ldots,t(\mu_{N-1})$ and $t(\mu_N)$. If $A(\mu)$ is equal to constant,the relationship between direct and isotropic diffuse radiation can be recovered [13,14].

## 3. Comparison results

In solar radiative transfer, it is important to perform the $\delta$-function adjustment to account for the forward-scattering peak [16]. The same is for the diffuse radiation. Here we examine the