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The borders of existence of anomalous convection flow in the inclined square cylinder: Numerical determination

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Abstract

The article is dedicated to the study of bifurcations of stationary convection regimes in a closed, heated from below and tilted square cylinder filled with air for cases of heat-insulated and perfectly heat-conducting sidewalls. The temperature and velocity fields were obtained using grid method for inclinations from a horizontal position up to 30 degrees in the range of Rayleigh numbers up to 20-fold excess of its critical value. The limit angle of anomalous-flow existence in the cylinder with the heat-insulated walls was established to be about 3 times greater than that in the cylinder with the heat-conducting ones. In the case of the heat-conducting walls the maximum angle of the anomalous-flow existence reached 7.7 degrees at a 3.3-fold excess of the critical value of Rayleigh number.

Keywords: Thermal convection; Inclination of the cavity; Anomalous flow; Numerical simulation.

Introduction

Thermal air convection in closed and tilted rectangular cavities is of interest due to the fact that containers of this type are the elements in a great number of technical devices. Their orientation can change smoothly or stepwise, while the convective flows in the gas filling the volume can undergo abrupt changes [1].

A cube is often used for simulating the effect of tilting on the convection modes in a closed rectangular cavity. At low and moderate Rayleigh numbers (Ra), convective air flows in the cube have the form of single-roll flows, i.e., of vortices with horizontal axes. Liquid particles in these flows are moving along circular paths in planes perpendicular to the vortex axis. Such a flow near the central vertical section of a cube can be considered quasi-two-dimensional [2]. This circumstance allows to expect a numerical study of 2D air flows, i.e., of infinitely extended horizontal vortices, in abstract infinite cylinders to provide insights into the observed bifurcation patterns of stationary convection regimes in laboratory experiments with a cubic cavity. The first numerical study on the influence of tilting (rotation of an infinite square cross-section cylinder around the axis) on the heat transfer between opposite isothermal walls (the other two walls

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were assumed to be heat-insulated) was carried out by Polezhaev [3]. This study established that the maximal heat flow was achieved at an intermediate tilt angle, i.e., between the heating from below and from the side.

The first data on the bifurcation of the convective air flow in a cubic cavity heated from below, caused by tilting, were published in the experimental study [4], which only considered small tilt angles, i.e., for the position corresponding to the heating strictly from below. It should be explained here that tilting at low Ra numbers results in the formation of a vortex with the circulation direction coinciding with the direction of the tilt angle of the cavity (if we regard the tilt angle as the rotation of the cavity from a zero angle). This vortex has normal circulation, and it stops rotating if the cavity is brought into a horizontal position. However, at Rayleigh numbers exceeding the critical value (Ra_c), it is possible for a vortex with a reverse circulation direction to coexist with the normal vortex. The term 'anomalous' was suggested for such flows in Ref. [5]. The directions of air circulation and of the cavity's tilt angle are opposite in an anomalous vortex, which means that the warm air flows downward along the tilted surface. Anomalous vortices exist within a certain range of tilt angles; the width of this range depends on the intensity of the convective flow. The experimental boundaries within which anomalous convective flow exists in a cube were experimentally determined in Ref. [6].

The goal of this study is in constructing a bifurcation curve reflecting the relationship between the critical tilt angle at which an anomalous vortex exists and the intensity of the convective flow.

The construction process should be based on numerically solving the full equations of thermal air convection (in the Boussinesq approximation) for different tilt angles of a square cavity and various critical parameter values.

Problem setting

Suppose that liquid fills a cavity shaped as an infinite horizontal cylinder of square cross-section (Fig. 1). Let us introduce a Cartesian coordinate system (x, y, z) whose y-axis coincides with an edge of the cylinder and is directed away from us. The unit vector $\mathbf{n}$ is located in the xz plane, points upward and is connected with the acceleration of gravity by the ratio $\mathbf{g}=-g\mathbf{n}$. The tilt angle of the square cylinder α is measured clockwise between the z-axis and $\mathbf{n}$. The variation range of the α angle in the calculations

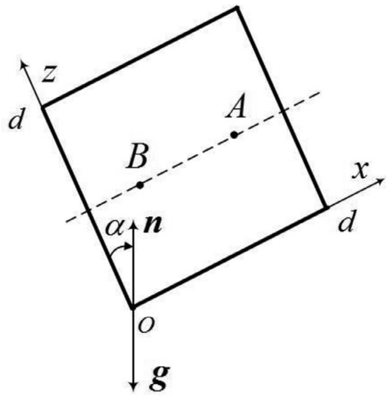

Fig. 1. Geometry of the problem on free thermal convection in a horizontal square cross-section cylinder.
The mean cross-section, marked by a dashed line, contains the points A and B between which the temperature difference dT is calculated. These points are located at a distance $d/4$ from the sidewalls (see the explanations in the text).

is $-30^\circ \leq \alpha \leq 30^\circ$; at $\alpha = 0^\circ$ the side of the cylinder coinciding with the x axis is horizontal, with the heating strictly from below. Fig. 1 shows the height-average square cross-section, with the points A and B marked; the temperature difference between these points is calculated for comparing the numerical results with the thermocouple measurements in a laboratory experiment [6].

The cavity walls are assumed to be solid. The upper and lower planes $z=0$, d are isothermal and maintained at a constant temperature difference Θ, with the $z=0$ plane the more heated. The calculations used two cavity models in which the sidewalls $x=0$, d are assumed to be either heat-conducting (a linear temperature distribution $T = \Theta(1 - z/d)$ is then given on them), or insulated (the equality $\partial T/\partial x = 0$ then describes the lack of heat flow through the surface). The linear expansion coefficient of the liquid β, the kinematic viscosity ν and the thermal diffusivity χ are constant.

It is assumed that the fluid is incompressible, and that the Boussinesq approximation holds true. The velocity v, the pressure p and the temperature T are determined by the continuity equations, by the Navier–Stokes, and the heat balance equations. Let us denote the distance, the temperature, and the flow and time functions as, respectively, d, Θ, the kinematic viscosity ν and d^2/ν. We shall seek for two-dimensional solutions of the problem. In this case, the vector fields of vorticity and flow function will differ from zero only in the y-components:

$$\vec{\varphi} = (0, \varphi, 0), \quad \vec{\psi} = (0, \psi, 0). \tag{1}$$