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On the initial state of the Universe in quantum cosmology

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Abstract

The paper discusses the problem of the initial state of a quantum inflationary Universe. Considering the dynamics of the inflation scalar field at the beginning of the inflation stage in the context of semi-classical approximation, we have identified this field with a cosmic time parameter. The early Universe state was defined as an initial value of the inflation field. Other degrees of the Universe freedom, including the scale factor, are treated within the framework of the quantum theory. The initial state of quantum degrees of freedom at the beginning of the inflation must be defined as well. A principle of the least excitation of physical degrees of freedom for the Universe has been proposed to determine the initial state of the quantum Universe. A uniform anisotropic model of the Universe was considered where its size and the anisotropic parameters were quantum dynamical variables. Assuming that the Universe size is a radial variable in the configuration space of the theory, the definition of the Hamiltonian of the Universe is rendered more precise. A simple exponential form of the initial state of the Universe is suggested and the Universe initial size is estimated for this form.

Keywords: Minimal excitation; Quantum Universe; Initial state; Anisotropic model.

1. Introduction

The Universe is currently understood to have been born from a geometrical point, i.e., essentially, from ‘nothing’, and consequently expanded enormously. This concept comes from classical relativistic cosmology based on Einstein’s general relativity (GR) as the consequence of Friedman’s solution of the theory’s equations. This solution is problematic for the classical theory because the matter density at the initial point is infinite. The Friedman model of the Universe is singular. Often this singularity is compared to that which arises in the Rutherford planetary atomic model where an electron moving on a cyclic orbit around the nucleus loses the energy and inevitably falls on it. There the electron energy becomes minus infinite.

It is well known that this singularity of the classical planetary model is resolved by quantum theory. Currently quantum theory seems equally promising in settling the problem of cosmological singularity. Although there is as yet no complete definition of the quantum version of GR, i.e. quantum cosmology, convincing scenarios for the evolution of the early Universe have been proposed. A number of theories discuss a quantum epoch in the emergence of the Universe from ‘nothing’, e.g., in the form of Vilenkin quantum tunneling [1], or of a similarly-defined Hartle–Hawking no-boundary wave function of the Universe [2].

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In the latter, the quantum epoch exists as a limitation on the semi-classical approximation of the no-boundary wave function in its continuation backward in time [3,4]. However, both theories predict a probability of initial values for a cosmic scalar field ϕ which "operates" the following expansion of the Universe. For a scalar field with a potential $V(\phi)$ the probability predicted, for instance, by the Vilenkin tunneling mechanism [5] is equal to

$$P(\phi) = \exp\left(-\frac{3}{8\hbar G^2 V(\phi_0)}\right), \tag{1}$$

where G is the Newton gravitational constant. The radius of the Universe "upon tunnel exit" equals

$$a_0(\phi_0) = \sqrt{\frac{3}{4\pi G V(\phi_0)}} \cong \frac{1}{2}\frac{1}{\sqrt{GV(\phi_0)}}. \tag{2}$$

The following stage of evolution is an exponential expansion of the Universe with the scalar field slow-rolling to its minimal value $\phi = 0$. This expansion is described by different inflation scenarios (see, for example, Linde [6]), where the initial radius of the Universe is taken to equal not its "tunneling" value (2) but the Planck radius $a_0 = l_{Pl}$. Still, the initial size of the Universe is not particularly important for its final state after its exponential expansion.

There is, however, a practical interest in calculating the quantum fluctuations of the inflaton scalar field which are assumed to be responsible for the inhomogeneities observed in the large-scale structure of the Universe. In any case, a quantum formulation of the inflationary phase of the evolution of the Universe seems necessary, especially at its early stage, when the inflaton scalar field ϕ is large and the radius of the Universe is small. Here we are faced with the fundamental problem of quantum cosmology, the absence of a time parameter and the impossibility of altogether formulating the quantum dynamics of the Universe [7]. The term "frozen dynamics" has been proposed to describe this specific case [8]. To "re-freeze" the dynamics, either a modification of quantum GR, or any physically motivated approximation that introduces a cosmic time parameter, is necessary.

Hosoya [9] proposed that the cosmic time during the inflationary phase in slow-roll of the inflaton scalar field ϕ with the potential

$$V(\phi) = \frac{\mu\phi^4}{4}.$$

could be the inflaton field itself (or, to be precise, its logarithm, $t \sim \ln(\phi/\phi_0)$, where ϕ_0 is the initial value of the field).

In this regime, the master wave equation of quantum cosmology, or the Wheeler–DeWitt (WdW) equation [7,8], takes (in a homogeneous model of the Universe) the form of a Schrödinger equation with the specified cosmic time parameter. In this approximation the radius of the Universe a is a quantum dynamical variable described by the wave function $\psi(t, a)$.

At this point the initial state of the Universe at $t = 0\,(\phi = \phi_0)$ must be determined. Ref. [9] regards the whole real axis (with negative values included) as the domain of definition of the scale variable a. The ordering of non-commuting operators in the WdW equation was chosen accordingly, and a Gaussian wave packet was taken as an initial state of the Universe. Before we can move on from discussing the quantum epoch in the birth of the Universe to the inflationary phase of its exponential expansion, we feel it is necessary to refine our definition of the initial state of the Universe.

First of all, let us limit the domain of definition of the scale factor: $a \in [0, \infty)$, assuming it to be a radial variable in the geometry sector of the configuration space of the Universe. The operator ordering in the WdW equation and the corresponding measure of integration in the configuration space must be selected accordingly. In this selection, we shall keep in mind the hyperbolic structure of the WdW equation for a more general anisotropic model of the Universe ("the mixmaster model") [8]. Let us define the initial state of the Universe in the present work as a state of minimal excitation of all physical degrees of freedom, including the anisotropy parameters. For this purpose let us use the principle of minimum energy for these excitations.

A generalization of the notion of energy in GR made by Witten as a positive definite functional for the case of an island mass distribution with the asymptotically flat space-time geometry [10,11] was discussed for the case of the closed Universe in [12].

For a maximally simplified homogeneous isotropic model of the Universe with none of the degrees of freedom but the inflaton scalar field taken into account in the present work, the minimum energy principle is reduced to that of its initial volume. Taking the mean values of the GR quantum constraint equations as additional conditions, we obtain the conditional principle for the initial state of the Universe.

2. Canonical quantization of a homogeneous anisotropic model of the Universe

Let us begin with a homogeneous anisotropic model of the Universe ("the mixmaster model" [8]), in which the 3D metrics of a spatial slice is parameterized as