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The trajectory analysis algorithm for electrostatic fields providing an angle focusing of given order in the plane of symmetry☆

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Abstract

A class of focusing electrostatic fields built as a solution for the inverse corpuscular optics problem has been investigated. An effective algorithm for trajectory analysis of these fields was suggested and tested. The algorithm was based on the special parametric form of potentials representation.

The main complexity of the problem is in treating the result of inverse form of potential representation, where coordinates are functions of the potential and the flux, but it is impossible to give the potential by an explicit function of coordinates. To solve the equations of motion in a direct form, it is necessary to find (numerically) coordinates at every integration step. It reduces the precision and increases the time of the calculations. We suggested using a parametric form of the potential and the relationship between coordinates. Direct equations of motion can be replaced with differential equations for parameters, which can be solved without any difficulty.

The results obtained can be applied to designing of new energy-analyzing devices with enhanced capabilities.

Keywords: Charged-particle energy analysis; Electron optics; Inverse problem; Angle focusing.

Solving inverse problems of charged particle dynamics in electrical and magnetic fields is a powerful tool for designing new corpuscular optics systems with the desired characteristics. Some of the noteworthy results obtained through this approach are the construction of an energy-analyzing system with ideal focusing in the symmetry plane [1], and the creation of mass-separating systems with ideal spatial and temporal beam focusing [2]. Even though a substantial range of analytical means has been accumulated this far (see, for example, Monographs [3,4]), not many inverse corpuscular optics problems have been solved [5], while obtaining and analyzing new solutions, and finding applications for them is without doubt of great interest and relevance.

The fields with a symmetry plane discussed in this study may serve as basis for constructing modern devices for analysis of variance of charged-particle beams by energy. The concept of such devices is in discriminating the particles by energy and concentrating them by other parameters (in particular, by starting angle). Recently, a number of monographs and papers [6–8] have been published developing the theoretical foundations for this type of devices, which attests to the current interest in energy analyzers.

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Polytechnic University professor Yu. K. Golikov suggested the following approach to setting the problem of searching for symmetrical electrostatic structures with the predetermined properties [1,5,9]. A *ZOX* plane is taken that is then associated with the system's symmetry plane. The potential *f* depends only on the coordinate *x*, the dependence *f*(*x*) is monotonic and provides the deceleration of a charged particle moving along the positive direction of the *x* axis. The particle drifts along the *z* axis, since the force is absent along the *z* coordinate. We assume below that a particle with the dimensionless energy *W* (the dimensionless variables are introduced according to [5,9]) starts from the origin of coordinates at an angle θ $(0<\theta<\pi/2)$ towards the *z* axis. After reflecting in the field *f*(*x*), the particle must again return to the *z* axis in the finish point $z=P(A)$, where $A=W\sin^2\theta$. The times of moving along the *x* and *z* coordinates must be equal, and this condition leads to an integral equation allowing to construct from the arrival function *P*(*A*) a one-dimensional implicit (as a dependence $x=F(f)$) potential distribution realizing the function *P*(*A*). The solution of this integral equation has the form

$$x=F(f)=\frac{1}{2\pi}\int\limits_0^f \frac{P(A)\,dA}{\sqrt{(f-A)(W-A)}} \tag{1}$$

and in some important cases can be written in elementary functions. In particular, the ideal angular focusing (in the angle range $0<\theta<\pi$) corresponds to the case $P(A)=1$,

$$x=\frac{1}{2\pi}\ln\frac{\sqrt{W}+\sqrt{f}}{\sqrt{W}-\sqrt{f}}=\frac{1}{2\pi}L. \tag{2}$$

If *f* is expressed through *x*, then we can obtain the well-known field of the Tutankhamun system [1,5,9]:

$$f(x)=\tanh^2\pi x. \tag{3}$$

A *k*th order focusing corresponds to the function

$$P(A)=1+g(A)(A-A_0)^{k+1},$$

where the value of the parameter A_0 is determined by the focusing angle θ_0, and $g(A)$ is the arbitrary dependence satisfying the condition $g(A_0)\neq 0$.

Let us list as an example the potentials implementing the above-described arrival function at $k=0,\ 1,\ 2$ and at $g(A)=1$:

$$x=F_0(f)=\frac{1}{4\pi}\Big(-2\sqrt{fW}-(f+W-2A_0+2)\,L\Big), \tag{4}$$

$$\begin{aligned}x=F_1(f)=\frac{1}{16\pi}\Big(&-2\sqrt{fW}(3\,(f+W)-8A_0)\\&-(3f^2+2fW+3W^2\\&-8A_0(f+W)+8{A_0}^2+8)\,L\Big),\end{aligned} \tag{5}$$

$$\begin{aligned}x=F_2(f)=\frac{1}{96\pi}\Big(&-2\sqrt{fW}(15\,(f^2+W^2)\\&+14fW-54A_0(f+W)+72{A_0}^2)\\&-(5\,(f^3+W^3)+3fW(f+W)\\&-12A_0fW-2A_0(9\,(f^2+W^2)\\&-12A_0(f+W)+8{A_0}^2)+16)\,L\Big),\end{aligned} \tag{6}$$

where *L* was determined earlier in formula (2). Potential (4) does not allow focusing.

For further comprehension, let us note a significant property of potentials (4)–(6). Choosing a polynomial arrival function

$$P(A)=1+(A-A_0)^{k+1},\quad {A_0}^{k+1}\neq 1,$$

results in an infinite derivative $x_f=\partial x/\partial f$ at $f=0$. Indeed, function (1), taking into account the chosen *P*(*A*), is a linear combination of the integrals

$$I_s=\int\limits_0^f\frac{A^s\,dA}{\sqrt{(f-A)(W-A)}},\quad s=0,\ 1,\ \dots,\ k+1$$

Obviously,

$$\frac{\partial I_0}{\partial f}=\frac{\sqrt{W}}{(W-f)\sqrt{f}}\xrightarrow[f\to 0]{}\infty.$$

It is easy to demonstrate that

$$\begin{aligned}\frac{\partial I_s}{\partial f}&=s\,I_{s-1}-\frac{1}{2}\sum_{m=0}^{s-1}W^m\,I_{s-1-m}\\&\quad+\frac{W^{k-1/2}\sqrt{f}}{W-f}\xrightarrow[f\to 0]{}0,\quad s>0,\end{aligned}$$

since for each non-negative s at $f\to 0$, $I_s\to 0$ as well. Correspondingly, the linear combination of integrals I_s, $s=0,\ 1,\ \dots,\ k+1$ is infinite at $f=0$ for non-negative integer *k*.

A possible question that may arise here is whether it is necessary to construct the fields implementing a finite-order focusing in the presence of a field providing ideal focusing. To address this, let us note that a lack of any parameters in the structure of field (3) does not allow optimizing the behavior of the beam (that is already far from perfect) in the plane orthogonal to the symmetry plane. At the same time, the controlled reduction in the focusing quality in the *ZOX*