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The sensitivity of the adaptive algorithm with a posteriori error control to marking criteria

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Abstract

The aim of this work is to compare different marking strategies, their influence on the work of adaptive algorithms with a posteriori error control for plane elasticity problems. The error control was performed using a functional error majorant. The implemented adaptive algorithms were based on the functional error majorant with no symmetry limitation on the free tensor, computed using the zero-order Raviart–Thomas approximations on triangular meshes. The four most commonly used element-marking criteria were used in adaptation. Numerical results for several plane-strain problems have been presented, including the case of different materials and geometry. A comprehensive analysis of the obtained results was given.

Keywords: A posteriori error estimate; Linear elasticity; Adaptive algorithm; Raviart–Thomas approximation.

1. Introduction

The paper discusses functional a posteriori error estimates for two-dimensional problems of linear elasticity theory. These estimates were first studied numerically in Ref. [1]. They were initially derived based on the relations of the duality theory of calculus of variations; this method was suggested in Ref. [2]. Later, Monograph [3] obtained the same estimates using the transformation of integral identities. Ref. [1] also discussed particular cases of estimates for a number of two-dimensional problems: plane strain, plane stress and axisymmetrical case.

The literature describes two types of functional error majorants for these problems: those explicitly and implicitly taking into account the symmetry of the free tensor that is a part of the estimate. Estimates of the second type allow using special finite elements developed for mixed methods. This approach was first suggested and implemented in Ref. [4].

Numerical studies of the functional approach to solving plane problems of linear elasticity theory were carried out by several authors. For example, Ref. [1] cites two examples of solving plane-strain problems with adapting the computational mesh in complexly shaped areas; in this case, the 'symmetrical' estimate and, respectively, the continuous piecewise-linear approximation of the finite-element method are used. The efficiency index of the estimate (i.e., the ratio of the error majorant to the estimated norm, the optimal index value is unity) increased, according to the results. The study [4] demonstrated that error overestimation increases for quadrilateral

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finite-element meshes: for some problems, the efficiency index exceeds the optimal value by about an order of magnitude on a mesh containing a total of several thousands of nodes. Refs. [4,5] used the zero-order Raviart–Thomas approximation and the Arnold–Boffi–Falk approximation with two additional degrees of freedom on each element for computing the functional majorants on nested quadrilateral meshes without adaptation. Aside from that, Ref. [6] studied the main theoretical properties and the aspects of practically implementing both types of functional a posteriori estimates, and listed the numerical results obtained by the adaptive algorithms for solving plane-strain problems. The theorems on the computational properties of the estimates and the corresponding error indicators have been formulated and proved.

The goal of this study is to perform a comparative analysis of various methods for selecting the elements for splitting (element marking) and the influence of these methods on the output of the mesh adaptation algorithm (the adaptive algorithm). Effectively, the study continues the research in [6] and takes as a basis some ideas from Monograph [7].

2. Problem setting

A plane problem of linear elasticity theory in the $\Omega \subset \mathrm{R}^2$ region with a Lipschitz -continuous boundary B consisting of two parts B_1 and B_2 has the form

$$\begin{cases} \sigma = L\varepsilon(u) & \text{in}\,\Omega \\ \mathrm{Div}\sigma + f = 0 & \text{in}\,\Omega \\ u = u_0 & \text{at}\,B_1 \\ \sigma n = F & \text{at}\,B_2 \end{cases}, \tag{1}$$

where Div is the tensor divergence.

The unknown is a vector displacement field $u(x_1,x_2)$ through which the strain tensor

$$\varepsilon(u) = \frac{1}{2}\left(\nabla u + (\nabla u)^T\right)$$

and the stress tensor σ are expressed.

The body force vector

$$f \in \mathrm{L}_2(\Omega, \mathrm{R}^2) = \mathrm{L}_2(\Omega) \times \mathrm{L}_2(\Omega),$$

the normal stresses $F \in \mathrm{L}_2(B_2,\mathrm{R}^2)$ on a part of the boundary B_2, and also the displacements $u_0 \in \mathrm{W}_2^1(\Omega,\mathrm{R}^2)$ at B_1 are given. The Lebesgue space L_2 is a space of square-integrable functions. The Sobolev space W_2^1 is a space of functions from L_2 whose generalized derivatives also belong to L_2. The vector n is the unit normal to B_2, L is the elastic constant tensor. It is assumed that there are positive constants λ_1 and λ_2 for the tensor L, such that

$$\lambda_1^2|\varepsilon|^2 \le L\varepsilon : \varepsilon \le \lambda_2^2|\varepsilon|^2 \tag{2}$$

for each tensor $\varepsilon \in \mathrm{M}_{sym}^{2\times 2}$, where $\mathrm{M}_{sym}^{2\times 2}$ is the space of symmetric second-order tensors of dimension 2. It is also assumed that the symmetry condition

$$L_{ijkm} = L_{jikm} = L_{kmij}, L_{ijkm} \in \mathrm{L}_\infty(\Omega),$$
$$i, j, k, m = 1, 2,$$

where the Lebesgue space $\mathrm{L}_\infty(\Omega)$ consists of functions bounded almost everywhere in Ω, is satisfied.

The solution of the problem (1) is sought for in the generalized sense:

Find the function u from $\mathrm{V} = u_0 + \mathrm{V}_0$, where $\mathrm{V}_0 = \{w \in \mathrm{W}_2^1(\Omega,\mathrm{R}^2)\,|\,w=0$ at $B_1\}$, satisfying the integral relation

$$\int_\Omega L\varepsilon(u) : \varepsilon(w)d\Omega = \int_\Omega f \cdot w d\Omega + \int_{B_2} F \cdot w dB \tag{3}$$

for any $w \in \mathrm{V}_0$.

Let $v \in \mathrm{V}$ be some approximate solution of the problem (3). In order to control the accuracy of the solution v, it is necessary to have an upper estimate for the energy norm

$$|||u - v||| := \left(\int_\Omega L\varepsilon(u - v) : \varepsilon(u - v)d\Omega\right)^{1/2}.$$

Ref. [1] obtained for the problem (1) a functional error majorant:

$$|||u - v||| \le C\left(\|\mathrm{Div}\tau + f\|_\Omega^2 + \|\tau n - F\|_{B_2}^2\right)^{1/2} + \\ + |||\tau_{sm} - L\varepsilon(v)|||_* + \frac{C_{\Omega B_1}}{\lambda_1}\|\tau_{sk}\|_\Omega, \tag{4}$$

where $\|...\|_\Omega$ and $\|...\|_{B_2}$ are the norms in L_2; τ is an arbitrary tensor from the Hilbert space

$$H = \left\{\begin{matrix} \tau \in \mathrm{L}_2(\Omega, \mathrm{M}^{2\times 2})| \\ \mathrm{Div}\tau \in \mathrm{L}_2(\Omega, \mathrm{R}^2), \\ \tau n \in \mathrm{L}_2(B_2, \mathrm{R}^2) \end{matrix}\right\};$$

τ_{sm} and τ_{sk} are the symmetric and the skew-symmetric parts of the tensor τ, respectively; $C_{\Omega B_1}$ is the constant from Korn's inequality that can be estimated numerically.

The auxiliary norm in the majorant is computed by the formula

$$|||\tau|||_* = \left(\int_\Omega L^{-1}\tau : \tau dx\right)^{1/2}.$$

The constant C must satisfy the inequality