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St. Petersburg Polytechnical University Journal: Physics and Mathematics 1 (2015) 278–286
www.elsevier.com/locate/spjpm

Stress singularity in a top of composite wedge with internal functionally graded material

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Available online 22 January 2016

Abstract

The antiplane problem of the composite wedge consisting of two homogeneous external wedge-shaped areas and an intermediate zone of the interphase is studied. The interphase material is assumed functionally graded. It is shown that the problem in each area is harmonic within the quadratic law of inhomogeneity of the material in the transverse direction. The influence of the interphase on the stress state at the top of the wedge is analyzed. As compared to the ideal contact of external materials, the presence of the interphase leads both to decrease and increase in the singularity exponent. Moreover, the stress asymptotic may have two singular terms for some values of the composite parameters.

Keywords: Antiplane crack; Functionally graded material; Interphase; Stress singularity.

Introduction

Stress state problems in the singular points of elastic bodies have been drawing the attention of researchers starting with the classical Williams study of 1952 [1]. These points may be angular, as well as positions where a change of boundary conditions or a discontinuity of elastic moduli in material occurs. The review in Ref. [2] offers an exhaustive bibliography on singular solutions of the problems of linear elastic fracture mechanics.

As opposed to a homogeneous medium where stresses exhibit a $r^{-\lambda}$ singularity in the crack tip ($\lambda = 1/2$, r is the distance from a crack tip), the order of stress singularities in an inhomogeneous medium may differ. For example, the stress singularity of a mode I or II interfacial crack with a perfect contact between the phases and the constant elastic moduli may exhibit an oscillating behavior. At the same time, the order of the singularity for an interfacial antiplane crack remains classical. However, if the contact between the materials is imperfect, the singularity will differ from the classical one, and it may be both strong ($1/2 < \lambda < 1$) and weak ($0 < \lambda < 1/2$) [3]. Moreover, for some models of imperfect contact, the asymptotic of the stresses will have two singular terms.

Strong and weak singularities existing for a crack in a two- or a three-phase medium under the conditions of the antiplane problem also follows from the results of Refs. [4,5]

The analysis of stress fields near the vertex of a composite wedge with piecewise-constant elastic prop-

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http://dx.doi.org/10.1016/j.spjpm.2015.11.006

erties and perfect phase contact was discussed in many studies, e.g., [4–9].

For some time, based on the conclusions in [10], the consensus regarding the functionally graded materials (FGM) whose elastic moduli vary continuously was that singularity exponent in the crack tip would always be the same as that in a homogeneous medium. However, in 2005, Carpinteri and Paggi showed [11] that for a crack growing in a gradient material with Young's modulus varying in the transverse direction the singularity exponent would differ from the classical value $\lambda = 1/2$. The exponential elastic modulus function of the polar angle was used to ensure the separation of variables in the differential equation.

A similar law of the shear module variation in the antiplane problem for a system of perfectly bonded wedges was used to construct singular solutions in Ref. [12]. Additionally, this article suggested an approximate method for determining the order of singularity in wedge-like areas, based on a piecewise-constant approximation of the shear module of FGM. However, the roots of the characteristic equation were not analyzed in Refs. [11,12].

The present work studies the stress state in the vertex of a composite wedge consisting of two homogeneous materials under the conditions of an antiplane problem. Instead of the traditional straight interface we examine a wedge-like FGM-filled area. We modeled the interface in this manner coming from the physical assumption that there is a diffusion of materials during their process connection [11,13,14]. This leads to the elastic modulus varying continuously; in contrast with Refs. [11,12] the modulus was assumed to depend quadratically on the polar angle in the transition area. This functional relationship for the shear module allows to obtain a characteristic equation of the problem in an explicit form and to analyze the equation roots causing the singularities depending on the composition parameters.

Problem statement

We are going to analyze the stress state of a composite wedge consisting of three wedge-shaped areas Ω_k $(k=1, 2, 3)$. The materials of two areas (Fig. 1):

$$\Omega_1 = \{(r,\theta) : 0 < r < \infty, \quad \beta < \theta < \alpha_1\},$$
$$\Omega_2 = \{(r,\theta) : 0 < r < \infty, -\beta < \theta < -\alpha_2\},$$

(r and θ are the polar coordinates) are considered homogeneous and isotropic with shear moduli μ_1 and μ_2, respectively.

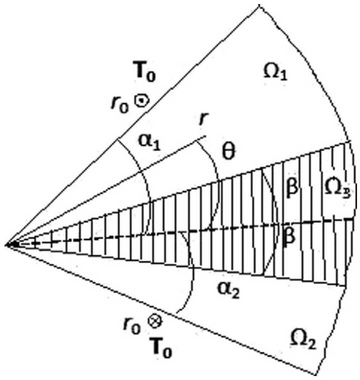

Fig. 1. A schematic for the setting of the problem on a composite wedge with a functionally graded interface (shaded): Ω_1, Ω_2, Ω_3 are the wedge-shaped areas; r, θ are the polar coordinates; $\boldsymbol{T}_0$ are the point forces applied to the wedge sides at a distance r_0 from its vertex; α_1, α_2, β are the geometrical parameters (angles) defining the area boundaries.

The geometrical parameters defining the boundaries of these areas must satisfy the following inequalities:

$$0 < \alpha_1 + \alpha_2 \le 2\pi, \quad 0 \le \beta \le \min(\alpha_1, \alpha_2).$$

The third (intermediate) area

$$\Omega_3 = \{(r,\theta) : 0 < r < \infty, -\beta < \theta < \beta\}$$

consists of an FGM that is modeled by an inhomogeneous material without accounting for its microstructure. The shear modulus of the functionally graded interphase μ_3 is assumed to depend only on the polar angle. The functional dependence $\mu_3(\theta)$ is such that the elastic modulus of the composite is continuous on the $\theta = \pm\beta$ boundaries, while its derivatives with respect to the angle θ have discontinuities on these boundaries.

The composite wedge is in equilibrium under antiplane deformation when subjected to self-equilibrating concentrated forces of magnitude T_0 applied to the wedge sides at a distance r_0 from its vertex (see Fig. 1). The contact of materials at the interface is assumed to be perfect.

From a mathematical standpoint, the problem is reduced to solving harmonic equations of equilibrium in each of the areas Ω_k $(k=1, 2)$:

$$\frac{\partial^2 w_k}{\partial r^2} + \frac{1}{r^2}\frac{\partial^2 w_k}{\partial \theta^2} + \frac{1}{r}\frac{\partial w_k}{\partial r} = 0. \qquad (1)$$

The shear stresses are found from the displacements w_k using the formulae

$$\tau_{\theta zk} = \frac{\mu_k}{r}\frac{\partial w_k}{\partial \theta}, \quad \tau_{rzk} = \mu_k \frac{\partial w_k}{\partial r}. \qquad (2)$$