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The development of a stratified flow following over a sphere inside the viscous fluid in the presence of internal or surface waves

Irina V. Stetsyuk*

Krylov State Research Centre, 44, Moskovskoe Shosse, St. Petersburg 196158, Russian Federation

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Abstract

This study has used the Large Eddy Simulation (LES) for numerical simulation of internal or surface waves. The viscous stratified flow over a sphere was investigated at the Reynolds and the Froude numbers $\mathrm{Re} = 2 \cdot 10^5$, $\mathrm{Fr} = 1.3$ for simulation of the flow over the sphere in the presence of internal waves, and at the internal Froude number $\mathrm{Fr}_i = 25$ for that in the presence of surface waves. The presence of background internal waves was found to result in an increase in the turbulent viscosity in the flow behind the sphere and in the vertical shift of the turbulent viscosity's maximum value. Moreover, their presence in the linearly stratified flow leads to a change in the density distribution of the near-surface layers of liquid. In this case the internal-wave breaking and wave mixing occur. The last one is caused by the interaction between the internal waves generated by the surface waves and the sphere.

Keywords: Numerical simulation; Stratification; Large eddy simulation; Flow over sphere; Viscous fluid.

1. Introduction

Calculating stratified flow properties is one of the problems of hydrodynamics of submerged bodies and it currently needs a solution taking into account the variation in the background characteristics of the flow. Some of these characteristics may include internal and surface waves as well as turbulence.

The interaction between the submerged bodies and the stratified fluid was studied theoretically [1,2] as well as experimentally [3] by many authors. A specific feature of stratified flows is that internal waves can propagate in them. When these collapse, turbulence is generated, with its scale depending on the properties of the initial internal wave [4]. Internal waves generated by a body moving in a fluid of non-uniform density are, in the general case, non-linear, and interact with the turbulence in the wake of a body. The characteristics of a non-uniformly dense flow in the wake of a body can be calculated by large-eddy simulation that takes into account the variability in the background conditions of the medium and their interaction with the turbulence in the body's wake. This method also

* Corresponding author. Tel.: +7 (812) 415-46-19; Fax: +7 (812) 415-45-90.
E-mail address: irene_stetsuok@yahoo.com.

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allows to take into account the turbulence anisotropy characteristic for stratified flows.

An external force acting vertically in the stratified fluid has an anisotropic effect on the flow structure. The nature of the turbulent fluctuation spectrum of the stratified flow is different from that of the uniform fluid. This is why modeling the turbulent flow around the body must significantly differ from the respective modeling of the flow with a uniform density field. In particular, the turbulence model must take into account the buoyancy-caused anisotropic effects [5].

Applying modern methods of computational fluid dynamics to studying the interaction between submerged objects and viscous stratified fluid opens up new possibilities for developing the hydrodynamics theory of bodies on internal waves.

2. Problem setting and numerical approximation

The following spatially averaged equations are used to describe the flow of viscous stratified fluid bounded by a free surface:the continuity equation of the form [6]

$$\frac{\partial \langle u_i \rangle_\Delta}{\partial x} = 0, \tag{1}$$

the Navier–Stokes equation

$$\frac{\partial \langle u_i \rangle_\Delta}{\partial t} + \langle u_j \rangle_\Delta \frac{\partial \langle u_i \rangle_\Delta}{\partial x_j} = -\frac{1}{\rho}\frac{\partial \langle p \rangle_\Delta}{\partial x_i} + \frac{\partial}{\partial x_j}\nu \frac{\partial \langle u_i \rangle_\Delta}{\partial x_j} - \frac{\partial \tau_{ij}^{SGS}}{\partial x_j} + g_i + F_\sigma, \tag{2}$$

the transport equation for the dimensionless density of the marine medium

$$\frac{\partial \langle f \rangle_\Delta}{\partial t} + \langle u_j \rangle_\Delta \frac{\partial \langle f \rangle_\Delta}{\partial x_j} = D_S \frac{\partial^2 \langle f \rangle_\Delta}{\partial x_j \partial x_j} - \frac{\partial J_j^{SGS}}{\partial x_j} \tag{3}$$

and the transport equations for the volume fraction of the marine medium, used for the volume of fluid method, of the form

$$\frac{\partial \langle \alpha_w \rangle_\Delta}{\partial t} + \langle u_j \rangle_\Delta \frac{\partial \langle \alpha_W \rangle_\Delta}{\partial x_j} = 0, \tag{4}$$

$$\rho = \rho_a + \alpha_W(\rho_l - \rho_a) + \alpha_W(\rho_h - \rho_l) f,$$
$$\nu = \nu_a + \alpha_W(\nu_l - \nu_a), \tag{5}$$

where u_i is a component of the fluid velocity vector, p is the pressure in the flow, ν is the kinematic fluid viscosity, D_S is the diffusion coefficient, g_i is a component of the gravitational acceleration vector,

$$\tau_{ij}^{SGS} = \widetilde{u_i u_j} - \tilde{u}_i \tilde{u}_j$$

is the subgrid-stress tensor,

$$F_\sigma = -\sigma \delta(\alpha_W) k n$$

is the surface tension body force of stratified fluid (σ is the surface tension coefficient; δ is the delta function determined at the seawater–air interface; k and n are the curvature and the normal to the interface surface),

ρ_a is the air density, ρ_l and ρ_h are the density values for a light and a heavy fluid,

$$f = (\rho - \rho_l)/(\rho_h - \rho_l)$$

is the dimensionless seawater density; brackets $\langle\rangle$ indicate averaging.

The subgrid stresses and scalar fluxes are parameterized using the Smagorinsky additional viscosity models [7]

$$\tau_{ij}^{SGS} = -2\nu_{SGS}\langle S_{ij} \rangle_\Delta,$$
$$J_j^{SGS} = -\frac{\nu_{SGS}}{\mathrm{Sc}_{SGS}}\frac{\partial \langle f \rangle_\Delta}{\partial x_j}, \tag{6}$$

where ν_{SGS} is the subgrid viscosity, Sc_{SGS} is the subgrid Prandtl–Schmidt number that in the general case depends on the Richardson number (Ri).

A modified mixing-length hypothesis taking into account the anisotropic effects due to buoyancy forces is used to find the ν_{SGS} value:

$$\nu_{SGS} = (C_S \Delta)^2 |S| f_S(\mathrm{Ri}). \tag{7}$$

Here C_S is the Smagorinsky constant, Δ is the filter width, $f_s(\mathrm{Ri})$ is the buoyancy function depending on the Richardson number [8]:

$$f_S(\mathrm{Ri}) = \left(1 - \frac{\mathrm{Ri}}{C_B}\right)^{3/2} (1 - \mathrm{Ri}/\mathrm{Sc}_{SGS})^{-1}, \tag{8}$$

where C_B is the constant ($C_B \approx 0.273$).

Wave propagation can be simulated numerically by imposing boundary conditions. It is at fluid interfaces that wave energy must be generated in order to solve the boundary value problem.

The system of Eqs. (1)–(5) should be complemented by initial and boundary conditions (listed below).

In the initial time, speed components are known

$$u_i(x_i, 0) = u_0, \tag{9}$$

as well as the pressure field

$$p = p_0 = \mathrm{const} \tag{10}$$

(where the constant is found from the hydrostatic law $\mathrm{const} = \rho g h$), the density field and the volume water fraction.