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Canonical decomposition of fluctuation interferences using the delta function formalism

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Abstract

The paper deals with the discrete spectral-orthogonal decompositions of centered Gaussian random processes for two cases. In the first case, the process implementations are a sequence of pulses that are short in comparison with the observation time. The process decomposition was obtained as a generalized Fourier series on the basis of the delta function formalism, and the variances of the coefficients (random values) of this series were found as well. The resulting expressions complement Kotel'nikov's formula because they cover both the high-frequency and the low-frequency regions of the canonical-decomposition spectrum. In the second case, a random process is a superposition of narrow-band Gaussian random processes, and its implementations are characterized by oscillations. For such a process the canonical decomposition in terms of the Walsh functions was obtained on the basis of the generalized function formalism. Then this decomposition was re-decomposed in terms of trigonometric functions; it follows from the resulting series that the canonical decomposition spectrum is not uniform since a pedestal is formed in the constant component region.

Keywords: Canonical decomposition; Narrow-band Gaussian process; Generalized function.

1. Introduction

The present work is dedicated to finding a canonical decomposition of fluctuation interferences affecting radio receivers; the implementation of a specific interference is given as a function on a finite carrier with either a fixed sign (Fig. 1(a)), or an alternating one (oscillating function) (Fig. 1(b)). It is well-established in the theory of stochastic processes that a canonical decomposition of a random process is its representation as a series consisting of products of random quantities and determinate time functions. This series converges to a mean square of the initial random process. A mathematical substantiation of the canonical decomposition of random functions was obtained by Karhunen and Loeve, as well as by Pugachev (the bibliography on this subject is listed in [1]).

The problem of the spectral decomposition of random processes discussed in this article is closely connected only with Ref. [2] where Kotel'nikov presented a decomposition to a Fourier series of normal fluctuation interferences acting during a 'sufficiently long' observation time. The interferences were represented as products of standardized Gauss uncorrelated random variables by

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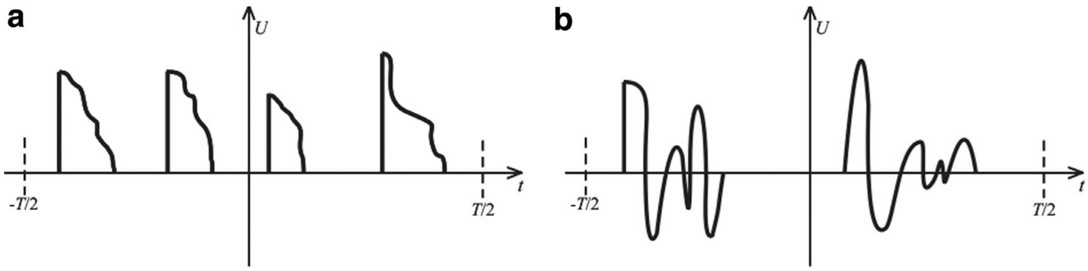

Fig. 1. A graphical representation of ways of defining a specific interference $U(t)$ in a time range $[-T/2, T/2]$ as a finite-carrier function: (a) with a fixed sign (positive in this example), (b) as an alternating function.

sines and cosines of multiple arguments with the principal period equal to the observation time. Let us keep in mind that Ref. [2] describes normal fluctuation interferences as some random positive-sign pulses that arrive at the input of radio systems owing to various natural factors (lightning discharges, etc.). In this case, the random character of the interferences is determined by three factors: the random time of pulses appearing, the random value of the area under the curve of each pulse of this type, and the random number of pulses in a fixed observation interval T. This latter interval sufficiently exceeds certain effective (mean) pulse duration. Kotel'nikov [2] found a spectral orthogonal decomposition of such a random process over the time interval of duration T with respect to a trigonometric basis using the central limit theorem (CLT) and the mean value theorem of integral calculus.

To apply the latter, each interference must be an implementation of a fixed-sign [2] continuous random function. If an auto-correlation function of such a random stationary process is given a priori as a delta function, then based on the well-known Wiener–Khinchin formula we shall obtain a constant value for the spectral density of the process power. This follows plainly from the canonical decomposition obtained by the author.

When it comes to the development of statistical radio engineering and radiophysics, Kotel'nikov should be given credit where credit is due; we should point out, for historical accuracy, that in his monograph [2] he obtained, among other calculations, an example of a canonical decomposition, even though the term 'canonical' itself entered the vocabulary of radio engineering specialists slightly later. O. Rice described a similar decomposition independent from Kotel'nikov, and virtually at the same time. Their radiophysics studies were then taken up by S.M. Rytov, L.A. Chernov, and other scientists, including the studies where the random part of the electron density was analytically defined for the purpose of solving the problem of wave propagation in randomly inhomogeneous media.

The first section of our study does not contain any new results; we essentially obtain, using another mathematical language, Kotel'nikov's calculations for the problem he considered in Chapter 2 of his monograph [2]. When obtaining the canonical decomposition we shall take the formalism of generalized functions and the central limit theorem as a basis. Let us assume that when the theorem is applied, either the Lyapunov [3,4] or Lindeberg's condition is fulfilled [4].

It is common knowledge that Lindberg's condition is fulfilled for a succession of independent and identically distributed random variables with finite variances [4]. It is our opinion that it is simpler to use the formalism of generalized functions than ordinary mathematical analysis [2].

In the second section, based on the delta-function formalism, we obtain a canonical decomposition of fluctuation interferences for the case when these interferences have an oscillating behavior. The sign of a physical quantity (e. g., voltage) corresponding to a specific interference changes more than once over the duration of the interference. This result is in our opinion new.

In the same section, without loss of generality, we examine narrowband random Gaussian interferences, or microbursts, and find the canonical decomposition for a sequence of these interferences over a sufficiently long period of time T. The approach using the mean value theorem taken in Ref. [2] is not suitable for this case due to the oscillating behavior of the random function under the integral.

However, the formalism of generalized functions allows to easily obtain an analytical representation of such a random process, with its canonical representation also being a white-noise decomposition. Importantly, the canonical decomposition in both sections is performed using the formalism of generalized functions.