Current Applied Physics 16 (2016) 1088–1093

Contents lists available at ScienceDirect

Current Applied Physics

journal homepage: www.elsevier.com/locate/cap

# Plasmonic-like waves on a chain of capacitively coupled rings of quantum dot tetramers

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ARTICLE INFO

*Article history:*
Received 1 March 2016
Received in revised form
2 June 2016
Accepted 11 June 2016
Available online 16 June 2016

*Keywords:*
Plasmonic-like wave
Quantum dot tetramer
Dispersion relation

ABSTRACT

In this paper, we consider a linear structure consisting of a specific array of capacitively coupled identical rings of quantum dot tetramers (QDT), where each ring consists of four, nearest neighbor tunnel-coupled, single-orbital quantum dots (QD) in the presence of Aharonov-Bohm (AB) magnetic flux. We show that due to a.c properties of QDT ring when subjected to AB flux, low frequency plasmonic-like waves can propagate on such a structure. Using exact diagonalization method, we, at first, determine the a.c responses and admittances between pairs of QDs in a single ring of QDT when threaded by AB flux. We then, using linear response theory, consider a specific array of identical QDT rings where each ring is coupled capacitively to its nearest neighbors. We present the necessary formulas and derive the equation for determining the dispersion relation and cut-off frequency. Finally, we numerically demonstrate the influence of the AB flux and the strength of coupling between the rings on the bandwidth of dispersion relation and its cut-off frequency.



## 1. Introduction

To engineer quantum circuits at nanoscale requires transmission of electromagnetic energy. One class of nanostructures which has been studied extensively for this purpose in the optical part of frequency spectrum is a linear chain of non-contacting spherical metallic nanoparticles [1–5]. The electromagnetic energy, below the diffraction limit, can propagate on such a structure via surface plasmon waves. But, due to the metallic nature and large number of electrons of the nanoparticles such a structure, besides being lossy [6], can not be used for low frequency parts of spectrum. Recently, a.c. properties of many mesoscopic systems consisting of quantum dots [7–11], quantum wires [12], mesoscopic rings [13–15] and etc. [16,17], have been studied both experimentally and theoretically. The quantum nature of these systems in coherent regime exhibits a spectrum of novel phenomena at low frequencies potentially viable for future applications in quantum circuits.

In this paper, based on the a.c. properties of a ring of QDT consisting of four nearest neighbor tunnel-coupled single-orbital quantum dots threaded by an external Aharonov-Bohm (AB) magnetic flux, we show that a specific ordered linear chain of such rings, with nearest neighbor capacitive coupling, can transmit low frequency signals via plasmonic-like waves with tunable cut-off frequency and bandwidth. Furthermore, if radiation loss is ignored, since each ring contains only four electrons, the transmission of plasmonic-like waves on such a structure is lossless, except close to transition frequencies of QDT.

The paper is organized as follows; In Sec. 2.1, the Hamiltonian of a single QDT ring and formalism of exact diagonalization method for determining its a.c. responses and admittances in the presence of AB flux are described. In Sec. 2.2, we present the model Hamiltonian and the necessary formulas for determining, respectively, the dispersion relations and normal mode frequencies of plasmonic-like waves of linear chains of infinite and finite numbers of QDT rings coupled together capacitively. In Sec. 3, we discuss our numerical results. Finally, in Sec. 4, we presented our conclusions. Technical details of some formulas are presented in appendices A, B and C.

## 2. Model and method

### 2.1. Single QDT ring

We consider a QDT ring consisting of four identical nearest neighbor tunnel coupled single-orbital QDs placed on the vertices

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http://dx.doi.org/10.1016/j.cap.2016.06.007
1567-1739/© 2016 Published by Elsevier B.V.

of a square with side length $d$ and Hamiltonian

$$H_R = \sum_{\substack{i=1\\ \sigma=\uparrow,\downarrow}}^{4} \varepsilon_i d_{i\sigma}^\dagger d_{i\sigma} + \sum_{\substack{<i,j>\\ \sigma=\uparrow,\downarrow}} \left( te^{\left(\frac{\pi i\phi}{2\phi_0}\right)} d_{j\sigma}^\dagger d_{i\sigma} + te^{\left(-\frac{\pi i\phi}{2\phi_0}\right)} d_{i\sigma} d_{j\sigma}^\dagger \right) + \frac{1}{2\varepsilon_R} \sum_{\substack{i,j\\ \sigma,\sigma'}} U_{\substack{i\,j\\ \sigma\,\sigma'}} n_{i\sigma} n_{j\sigma'}, \tag{1}$$

where $d_{i\sigma}$ and $d_{i\sigma}^\dagger$ are annihilation and creation operators of electron with spin $\sigma$ at the $i$-th QD, $n_{i\sigma} = d_{i\sigma}^\dagger d_{i\sigma}$, is the electron number operator with spin $\sigma$, $t$ is the hopping integral, $\phi$ is the AB magnetic flux, $\phi_0$ is the quantum flux, $U_{\substack{ii\\ \uparrow\downarrow}} = U_0$, $U_{\substack{\langle ij\rangle\\ \sigma\sigma'}} = \frac{U_0}{2}$, and $U_{\substack{\langle\langle ij\rangle\rangle\\ \sigma\sigma'}} = \frac{U_0}{2\sqrt{2}}$ are the strength of electron-electron interactions for on-site, nearest neighbors, $\langle ij\rangle$, and next nearest neighbors$\langle\langle ij\rangle\rangle$, respectively, and $\varepsilon_R$ is the static dielectric constant of the medium.

The *a.c* admittances, in the time domain, $G_{ij}(t-t')$, between any two QDs of the QDT ring, are related to the retarded density-density response functions, $\chi_{ij}^R(t-t')$, by the following relation;

$$G_{ij}(t-t') = -\frac{e^2}{\hbar}\frac{d}{dt}\chi_{ij}^R(t-t'), \quad i,j = 1,\cdots,4, \tag{2}$$

where

$$\chi_{i,j}^R(t-t') = -i\theta(t-t') \sum_{\sigma,\sigma'} \left\langle \Psi_0 \middle| \left[ n_{H,i\sigma}(t), n_{H,j\sigma'}(t') \right] \middle| \Psi_0 \right\rangle \tag{3}$$

In the above equation $n_{H,i\sigma}(t)$ s are the aforementioned electron number operators in the Heisenberg representation, $|\Psi_0\rangle$ is the exact ground state of the Hamiltonian, $e$ is the charge of electron and $\hbar$ is the planck constant. Furthermore, due to symmetry of QDT, we have $\chi_{ij}^{(R)}(\omega) = \chi_{ji}^{(R)}(\omega)$. Using the definition of Heisenberg representation, Eq. (3) can be written in the following way

$$\chi_{i,j}^R(t-t') = -i\theta(t-t') \sum_m \sum_{\sigma\sigma'} \left\{ \left\langle \Psi_0 \middle| e^{-\frac{iHt}{\hbar}} n_{i\sigma} e^{\frac{iHt}{\hbar}} \middle| \Psi_m \right\rangle \left\langle \Psi_m \middle| e^{-\frac{iHt'}{\hbar}} n_{j\sigma'} e^{\frac{iHt'}{\hbar}} \middle| \Psi_0 \right\rangle - \left\langle \Psi_0 \middle| e^{-\frac{iHt'}{\hbar}} n_{j\sigma'} e^{\frac{iHt'}{\hbar}} \middle| \Psi_m \right\rangle \left\langle \Psi_m \middle| e^{-\frac{iHt}{\hbar}} n_{i\sigma} e^{\frac{iHt}{\hbar}} \middle| \Psi_0 \right\rangle \right\} \tag{4}$$

where $i,j = 1 \ldots,4$ and $\{|\Psi_m\rangle\}$ is a complete set of energy eigenstates of the Hamiltonian with eigenenergies $\{E_m\}$. Fourier transforming Eq. (4) into frequency domain, we have

$$\chi_{i,j}^R(\omega) = \hbar \sum_m \sum_{\sigma\sigma'} \left[ \frac{\langle \Psi_0 | n_{i\sigma} | \Psi_m \rangle \langle \Psi_m | n_{j\sigma'} | \Psi_0 \rangle}{\hbar\omega - (E_m - E_0) + i\eta\hbar} - \frac{\langle \Psi_0 | n_{j\sigma'} | \Psi_m \rangle \langle \Psi_m | n_{i\sigma} | \Psi_0 \rangle}{\hbar\omega + (E_m - E_0) + i\eta\hbar} \right], \quad i,j = 1,\cdots,4. \tag{5}$$

We determine $\{|\Psi_m\rangle\}$ and $\{E_m\}$ using the exact diagonalization method [18]. In this method, the fermionic annihilation and creation operators are represented by their matrix form with respect to a basis constructed from the tensor product of all possible states of each QD, $\{|0\rangle_i, |\uparrow\rangle_i, |\downarrow\rangle_i, |\uparrow\downarrow\rangle_i\}$. For a QDT ring consisting of four QDs, this is a $4^4$-dimensional matrix representations of the fermionic creation and annihilation operators which must also satisfy the anti-commutation relations of the field operators. The general form of the annihilation and creation operators in this representation are [18].

$$a_{i\sigma} = \underbrace{P\otimes\cdots\otimes P}_{i-1} \otimes Q_\sigma \otimes \underbrace{I\otimes\cdots\otimes I}_{4-i}, \quad i = 1,\cdots,4. \tag{6}$$

and

$$a_{i\sigma}^\dagger = \underbrace{P\otimes\cdots\otimes P}_{i-1} \otimes Q^\dagger \otimes \underbrace{I\otimes\cdots\otimes I}_{4-i}, \quad i = 1,\cdots,4. \tag{7}$$

where $I$ is a $4 \times 4$ unit matrix and $P$, $Q_\sigma$, $Q_\sigma^\dagger$ are $4 \times 4$ matrices which they are given in the App.A.

By diagonalizing the Hamiltonian in this representation, we can obtain all the many-body eigenstates of the QDT ring with their respective eigenenergies. In Fig. 1 the first six eigenenergies of four-electron subspace as functions of AB flux are depicted for $\frac{U_0}{\varepsilon_R t} = 0.2$ and onsite energies $\varepsilon_i = -\frac{U_0}{2} - 2U_1 - U_2$ , $i = 1,\cdots,4$ where the Hamiltonian, $H_R$, is particle-hole symmetric. In Fig. 2, the real parts of $\chi_{11}^R(\omega), \chi_{12}^R(\omega)$ and $\chi_{13}^R(\omega)$ as functions of $\frac{\hbar\omega}{t}$ for $\phi = 8.064 \times 105$ G$-$(nm)$^2$ are given. In Fig. 3 the low frequency parts of the aforementioned response functions for $\phi = 2.688 \times 105$ and $1.075 \times 106$ G$-$(nm)$^2$ vs. frequency are depicted, and in Fig. 4, the real parts of $\chi_{11}^R(0), \chi_{12}^R(0)$ and $\chi_{13}^R(0)$ are presented as functions of AB magnetic flux, $\frac{\phi}{\phi_0}$. It should be noted that in all cases presented in the above figures, $\chi_{11}(\omega)$ and $\chi_{13}(\omega)$ have a same signe and opposite to $\chi_{12}(\omega)$.

### *2.2. Chain of QDT rings*

In this section, we consider the linear structure, represented in Fig. 5. We assume that each ring is threaded with AB flux, and embedded in a dielectric medium with static dielectric constant $\varepsilon_R$ which can be different from the static dielectric constant of surrounding medium which we assume it is equal to one. The Hamiltonian for the specific arrangement of rings in Fig. 5 is

$$H_{SB} = \sum_l H_R^{(l)} + H_C, \tag{8}$$

where $H_R^{(l)}$ is the Hamiltonian of the l-th QDT ring, given by Eq. (1), and $H_c$ represents the capacitive coupling of QDT rings which for the configuration of Fig. 5 has the form

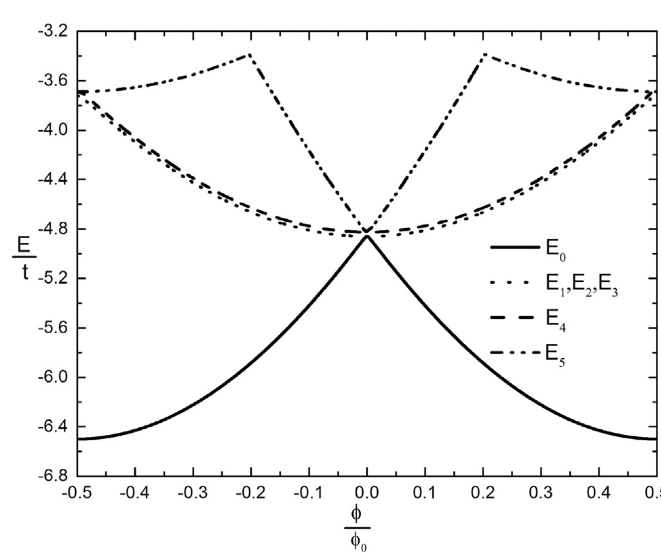


**Fig. 1.** The six lowest eigenenergis of a QDT ring, consisting of four electrons, in unit of $t$ as a function of AB magnetic flux for $\frac{U_0}{\varepsilon_R t} = 0.2$.