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The nano scale vibration of protein microtubules based on modified strain gradient theory

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ABSTRACT

This paper investigates the free vibration of protein microtubules (MTs) embedded in the cytoplasm by using linear and nonlinear Euler–Bernoulli beam model based on modified strain gradient theory. The protein microtubule is modeled as a simply support or clamped–clamped beam. Beside, the elastic medium surrounding of MTs is modeled with Pasternak foundation. Vibration equations are obtained by using Hamilton principle and these equations are solved according to boundary conditions. Finally the dependency of vibration frequencies on environmental conditions, MTs size, changes of temperature and material length scale parameters (size effects) is studied. By comparing the findings, it could be said that the MTs' frequency is greatly increased in the presence of cytoplasm and it is very dependent to material length scale parameters.

1. Introduction

Cytoplasm is an important part of eukaryotic cells made up of three types of fibers called protein microtubules, actin filaments and intermediate filaments. Based on empirical studies, protein microtubules are 100 times stronger than actin and intermediate filaments and provide strength of cell. Furthermore, MTs perform vital role in biological functions such as cell division, intracellular transport and separation of chromosomes during mitosis. Thus to identify the behavior of cells, understanding the mechanical characterization of MTs is important. Structurally, MTs are made up of lateral connection of long fibers named protofilaments whose number ranges from 8 to 20 in different MTs. Protofilaments are composed of successive bonds between α and β tubulins. Geometrically, MTs are in the shape of hollow cylinders with outer and inner diameters of about 25 and 15 nm, respectively, and whose length ranges from 10 nm to 100 μm. In order to study MTs' mechanical behavior, they are considered cylindrical with an equivalent thickness of 2.7 nm [1–5]. Of course their effective thickness in bending equals 1.6 nm [6]. So far, many mathematical and empirical researches have been conducted on MTs mechanical properties. Some researchers have studied the mechanical behavior of MTs regardless of size effects, based on classical theory of elasticity [7–10]. Other researchers such as Tounsi et al. [11] used higher-order shear deformation theory for analyzing the length-dependent flexural rigidity of protein microtubules but it is obvious that, classical theory is incapable of analyzing the effects of length scale parameters. On the other hand, in micro and nano scales, size effects could not be ignored. Therefore, in analyzing these structures, higher-order continuum theories which include size effect parameters must be used. These theories include some theories such as: nonlocal Eringen theory [12], Cosserat theory [13], Couple stress theory [14] and strain gradient theory [15]. By modifying the strain gradient theory, Lam et al. [16] offered modified strain gradient theory which includes three length scale parameters. Using this theory, for the first time Akgöz and Civalec [17,18] studied the buckling of MTs. Many studies have been conducted to investigate size effects on MTs' vibration, using nonlocal Euler–Bernoulli beam model [19]. Civalek et al. [20] explored linear vibration of MTs Based on nonlocal elasticity and using Euler–Bernoulli beam model. Heireche et al. [21] studied MTs vibrations using nonlocal elasticity and Timoshenko beam model. Shen [22] studied the nonlinear vibration of MTs by using nonlocal shear deformable cylindrical shell model and considering the surrounding elastic medium. He showed that, nonlinear MTs' frequency increased in the present of Pasternak foundation.

As mentioned above, so far several studies have been conducted to examine the vibration behavior of MTs but none of them have used the strain gradient theory. Therefore, in this study, for the first time, vibration behavior of MTs is examined by using strain

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gradient theory. In order to analyze the mechanical characteristics of beam-like structures, both linear and nonlinear analyses are needed. If the ratio of lateral deflection to beams thickness is smaller than one, linear analysis will be used. While this ratio is equal to or greater than one, nonlinear effects appear and therefore, nonlinear models must be used [23,24].

In the present study, based on the modified strain gradient theory, and by using the linear and nonlinear Euler–Bernoulli beam models, effects of the surrounding medium, MTs sizes, temperature changes and size effect parameters have been studied in the MTs vibration. By comparing the results, it is observed that the MTs' frequency is greatly increased in the presence of cytoplasm and it is very dependent to material length scale parameters.

2. Strain gradient formulation of MTs nonlinear vibration equation

Based on modified strain gradient theory the strain energy per volume unit is given as follows [23]:

$$\overline{U} = \frac{1}{2}\left(\sigma_{ij}\varepsilon_{ij} + m_{ij}^{(s)}\chi_{ijk}^{(s)} + \eta_{ijk}^{(1)}\tau_{ijk}^{(1)} + p_i\gamma_i\right). \tag{1}$$

In above equations, σ_{ij} is the Cauchy stress tensor, ε_{ij} is the strain tensor, χ_{ij} is the symmetric part of the rotation gradient tensor and γ_i and η_{ijk} are the dilatation gradient vector and the deviatoric stretch gradient tensor, respectively. Also, m_{ij}, p_i and τ_{ijk} are the higher-order stresses. These tensors and vectors are defined as follows:

$$\varepsilon_{ij} = \frac{1}{2}\left(u_{i,j} + u_{j,i}\right), \tag{2}$$

$$\chi_{ij}^{(s)} = \frac{1}{2}\left(\theta_{i,j} + \theta_{j,i}\right), \tag{3}$$

$$\theta_i = \frac{1}{2}e_{ijk}u_{k,j}, \tag{4}$$

$$\gamma_i = \varepsilon_{mm,i}, \tag{5}$$

$$\begin{aligned}\eta_{ijk}^{(1)} = {} & \frac{1}{3}\left(\varepsilon_{jk,i} + \varepsilon_{ki,j} + \varepsilon_{ij,k}\right) - \frac{1}{15}\Big[\delta_{ij}\left(\varepsilon_{mm,k} + 2\varepsilon_{mk,m}\right) \\ & + \delta_{jk}\left(\varepsilon_{mm,i} + 2\varepsilon_{mi,m}\right) + \delta_{ki}\left(\varepsilon_{mm,j} + 2\varepsilon_{mj,m}\right)\Big],\end{aligned} \tag{6}$$

$$\sigma_{ij} = \lambda\varepsilon'_{kk}\delta_{ij} + 2\mu\varepsilon'_{ij}, \quad \varepsilon'_{ij} = \varepsilon_{ij} - \alpha\Delta T, \quad \Delta T = T - T_0, \tag{7}$$

$$m_{ij}^{(s)} = 2l_2^2\mu\chi_{ij}^{(s)}, \tag{8}$$

$$p_i = 2\mu l_0^2\gamma_i, \tag{9}$$

$$\tau_{ijk}^{(1)} = 2\mu l_1^2\eta_{ijk}^{(1)}. \tag{10}$$

In above equations, α is thermal expansion coefficient, u_i is the displacement vectors, θ_i is the infinitesimal rotation vector, T_0 is environmental temperature and l_0, l_1 and l_2 are material length scale parameters. Also λ and μ are Lame constants which are defined as follows:

$$\lambda = \frac{\upsilon E}{(1+\upsilon)(1-2\upsilon)}, \quad \mu = \frac{E}{2(1+\upsilon)}. \tag{11}$$

Here, E and υ are Young's modulus and Poisson's ratio of MTs, respectively. Based on the modified strain gradient theory and considering the nonlinear effects, MTs' total potential energy U is measured as follows:

$$\begin{aligned}U &= \int_V\left(\frac{P_0}{A}\varepsilon_{11} + \overline{U}\right)\mathrm{d}A\mathrm{d}x \\ &= \frac{1}{2}\int_V\left(2\frac{P_0}{A}\varepsilon_{11} + \sigma_{ij}\varepsilon_{ij} + m_{ij}^{(s)}\chi_{ij}^{(s)} + \eta_{ijk}^{(1)}\tau_{ijk}^{(1)} + p_i\gamma_i\right)\mathrm{d}A\mathrm{d}x.\end{aligned} \tag{12}$$

In the above integral, the first part represents the potential energy caused by external axial force P_0. Indeed P_0/A is the initial axial stress in the beam assumed to have consistent distribution in the section of the beam. Fig. 1 shows the simply supported (S–S) or clamped–clamped (C–C) beam model for MTs embedded in the cytoplasm in which X, Y and Z-axes represent direction along the length, width and thickness of beam, respectively. Afterward, using of calculus variation method, dynamic equations of MTs are produced. In the present study, the Euler–Bernoulli beam model is used whose displacement components are as follows:

$$u_1 = u_0 - z\frac{\partial w}{\partial x}, \quad u_2 = 0, \quad u_3 = w(x,t). \tag{13}$$

In Equation (13) u_1, u_2 and u_3 represent displacement components along the X, Y and Z-axes, respectively. Here u_0 is the initial displacement of the middle plane along X-axis, and z represents the distance of each point on the beam section from the middle plane along the Z-axis. In nonlinear analysis, in order to analyze the geometrical nonlinearity, Von-Karman strain model has been used. By inserting Equation (13) into Equation (2), according to the Von-Karman strain model, the only nonzero strain is obtained as follows:

$$\varepsilon_{11} = \frac{\partial u_0}{\partial x} + \frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2 - z\frac{\partial^2 w}{\partial x^2}. \tag{14}$$

Similarly, by inserting Equation (2) into Equations (3)–(6), the nonzero components of the symmetric part of the rotation gradient tensor, the dilatation gradient vector and the deviatoric stretch gradient tensor will be obtained (for detail see Appendix A) and replacing Equations (14) and (A.1)–(A.3) into relations (7)–(10), the nonzero values of classic and higher-order stress tensors are obtained (for detail see Appendix A). By replacing Equations (14) and (A.1)–(A.7) into Equation (12), after simplification, the total potential energy of MTs is obtained as follows:

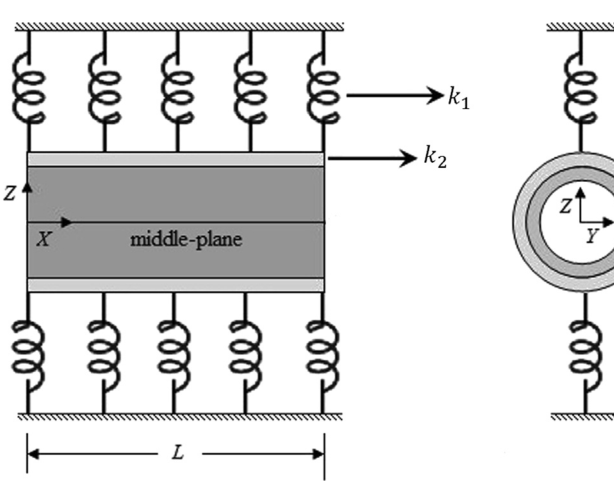

Fig. 1. Schematics beam model of MTs inside the cytoplasm.