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An accurate and robust numerical method for micromagnetics simulations

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ABSTRACT

We propose a new robust, accurate, and fast numerical method for solving the Landau–Lifshitz equation which describes the relaxation process of the magnetization distribution in ferromagnetic material. The proposed numerical method is second-order accurate in both space and time. The approach uses the nonlinear multigrid method for handling the nonlinearities at each time step. We perform numerical experiments to show the efficiency and accuracy of the new algorithm on two- and three-dimensional space. The numerical results show excellent agreements with exact analytical solutions, the second-order accuracy in both space and time, and the energy conservation or dissipation property.

1. Introduction

The Landau–Lifshitz (LL) equation [1] which describes the evolution of the magnetization in a ferromagnetic material [2,3] plays an important role in understanding the mechanisms of magnetization [1,4].

In one-dimensional case, many authors have studied the soliton solution, the interaction of solitary waves, and other properties of the solitary waves [5–7]. Also, the high-dimensional dynamics have been researched in Refs. [8–11].

In this paper, we consider a new robust and accurate numerical method for the Landau–Lifshitz equation with a damping term:

$$\frac{\partial \mathbf{m}(\mathbf{x},t)}{\partial t} = -\mathbf{m}(\mathbf{x},t)\times\Delta\mathbf{m}(\mathbf{x},t) - \mu\mathbf{m}(\mathbf{x},t)\times[\mathbf{m}(\mathbf{x},t)\times\Delta\mathbf{m}(\mathbf{x},t)], \tag{1}$$

where $\mathbf{m}(\mathbf{x},t) = (u(\mathbf{x},t), v(\mathbf{x},t), w(\mathbf{x},t))$ is a magnetization vector field for $\mathbf{x}\in\Omega$ and $0 < t \le T$. Here, $\mu \ge 0$ is the damping parameter and $\Omega \subset \mathbb{R}^d$ $(d = 1,2,3)$ is a domain.

In this paper, we consider the simplified LL equation which is not magnetostatic, anistotropy, and Zeeman field. However, we note that this simplification does not limit the proposed analysis.

Here, we review briefly the properties of the Landau–Lifshitz equation:

- Let $E(\mathbf{m}(t)) := \int_\Omega |\nabla\mathbf{m}(\mathbf{x},\ t)|^2 d\mathbf{x}$ be an energy. Here, if $\mathbf{x} = (x,y,z)$, then $|\nabla\mathbf{m}(\mathbf{x},t)|$ is defined as

$$\left|\nabla\mathbf{m}(\mathbf{x},t)\right| = \sqrt{\left|\frac{\partial\mathbf{m}(\mathbf{x},t)}{\partial x}\right|^2 + \left|\frac{\partial\mathbf{m}(\mathbf{x},t)}{\partial y}\right|^2 + \left|\frac{\partial\mathbf{m}(\mathbf{x},t)}{\partial z}\right|^2},$$

where $\frac{\partial\mathbf{m}(\mathbf{x},t)}{\partial x} = \frac{\partial u(\mathbf{x},t)}{\partial x}\mathbf{i} + \frac{\partial v(\mathbf{x},t)}{\partial x}\mathbf{j} + \frac{\partial w(\mathbf{x},t)}{\partial x}\mathbf{k}$, and so on. To represent it as the other form of energy $E(\mathbf{m}(t))$, we take a derivative $\partial\mathbf{m}/\partial t$ as

$$\frac{\partial E(\mathbf{m}(t))}{\partial t} = \int_\Omega \left(2\frac{\partial\mathbf{m}}{\partial x}\cdot\frac{\partial^2\mathbf{m}}{\partial t\partial x} + 2\frac{\partial\mathbf{m}}{\partial y}\cdot\frac{\partial^2\mathbf{m}}{\partial t\partial y} + 2\frac{\partial\mathbf{m}}{\partial z}\cdot\frac{\partial^2\mathbf{m}}{\partial t\partial z}\right)d\mathbf{x}.$$

Integration by parts using the zero Neumann or periodic boundary conditions gives

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$$\frac{\partial E(\mathbf{m}(t))}{\partial t} = \left(2\frac{\partial \mathbf{m}}{\partial x}\cdot\frac{\partial \mathbf{m}}{\partial t} + 2\frac{\partial \mathbf{m}}{\partial y}\cdot\frac{\partial \mathbf{m}}{\partial t} + 2\frac{\partial \mathbf{m}}{\partial z}\cdot\frac{\partial \mathbf{m}}{\partial t}\right)\Big|_{\partial\Omega} - 2\int_\Omega \left(\frac{\partial^2 \mathbf{m}}{\partial x^2}\cdot\frac{\partial \mathbf{m}}{\partial t} + \frac{\partial^2 \mathbf{m}}{\partial y^2}\cdot\frac{\partial \mathbf{m}}{\partial t} + \frac{\partial^2 \mathbf{m}}{\partial z^2}\cdot\frac{\partial \mathbf{m}}{\partial t}\right)d\mathbf{x}$$
$$= -2\int_\Omega \Delta\mathbf{m}\cdot\mathbf{m}_t d\mathbf{x} = 2\int_\Omega \Delta\mathbf{m}\cdot(\mathbf{m}\times\Delta\mathbf{m} + \mu\mathbf{m}\times[\mathbf{m}\times\Delta\mathbf{m}])d\mathbf{x} = 2\int_\Omega \Delta\mathbf{m}\cdot(\mu\mathbf{m}\times[\mathbf{m}\times\Delta\mathbf{m}])d\mathbf{x}$$
$$= 2\int_\Omega (\mu\mathbf{m}\times[\mathbf{m}\times\Delta\mathbf{m}])\cdot\Delta\mathbf{m}d\mathbf{x} = 2\mu\int_\Omega (\Delta\mathbf{m}\times\mathbf{m})\cdot(\mathbf{m}\times\Delta\mathbf{m})d\mathbf{x} = -2\mu\int_\Omega (\mathbf{m}\times\Delta\mathbf{m})\cdot(\mathbf{m}\times\Delta\mathbf{m})d\mathbf{x} = -2\mu\int_\Omega |\mathbf{m}\times\Delta\mathbf{m}|^2 d\mathbf{x}.$$

By integration, we obtain the following energy equation

$$E(\mathbf{m}(t)) = E(\mathbf{m}(0)) - 2\mu\int_0^t\int_\Omega \left|\mathbf{m}(\mathbf{x},s)\times\Delta\mathbf{m}(\mathbf{x},s)\right|^2 d\mathbf{x}ds \qquad (2)$$

for any $t > 0$. Equation (2) implies that this problem has energy dissipation property for the case $\mu > 0$ and energy conservation property for the case $\mu = 0$.

- Equation (1) has length-preserving property, i.e., $|\mathbf{m}(\mathbf{x},t)| = |\mathbf{m}(\mathbf{x},0)|$ for any $t > 0$. To show this, we do scalar multiplication of Equation (1) with $\mathbf{m}$,

$$\frac{\partial \mathbf{m}}{\partial t}\cdot\mathbf{m} = -(\mathbf{m}\times\Delta\mathbf{m})\cdot\mathbf{m} - \mu\mathbf{m}\times(\mathbf{m}\times\mathbf{m})\cdot\mathbf{m} = 0.$$

Then, $\partial\left|\mathbf{m}\right|^2/\partial t = 0$, which implies $|\mathbf{m}(\mathbf{x},t)|$ is constant for all t and each $\mathbf{x}$, that is, $|\mathbf{m}(\mathbf{x},t)| = |\mathbf{m}(\mathbf{x},0)|$. And we assume that $|\mathbf{m}(\mathbf{x},0)| = 1$. For a more detailed discussion of the model, see survey articles [12–18].

Numerical method has become an important tool in the study of dynamics of ferromagnetic materials [19–22].

Since explicit methods cause severe time step restriction for stability [23], several methods such as the semi-analytical schemes [23,24] and the high order Runge–Kutta algorithms [25] have been proposed to improve their efficiency. A geometric integration technique based on Cayley transform is applied to the time discretization of the LLG equation [26].

Through the finite element procedure [27–29], numerical solutions are obtained by the extrapolation formula leading to semi-implicit [27,29]. Otherwise, iterative techniques, as fixed-point [30] and quasi-Newton algorithms [31], are needed.

The midpoint rule time discretization technique was applied to Landau–Lifshitz–Gilbert equation [31]. The Gauss–Seidel projection method (GSPM) was introduced by Wang et al. [32] and the improved GSPM was presented by Garcia–Cervera and Weinan [33]. A successive over relaxation method was presented for the LLG [34].

Jeong and Kim [35] suggested a Crank–Nicolson scheme which is accurate, however, it uses an updated source term and repeatedly performed iterations until the numerical solution converges. In this paper, we propose a new robust fast accurate numerical method for computations of the LL equation. The proposed method does not need an updated source term and therefore it is fast. We also perform three-dimensional space experiments.

The contents of this paper are as follows. In Section 2, we describe the discrete semi-implicit finite difference scheme of the Landau–Lifshitz equation. Numerical experiments such as a second-order convergence test and an energy conservation property of the proposed scheme are given in Section 3. In Section 5, conclusions are drawn.

2. Numerical solution

For simplicity of exposition, we shall first discretize the LL equation in one dimensional domain $\Omega = (0,1)$ with a uniform grid with the number of grid points N_x, a space step $h = 1/N_x$, and a time step $\Delta t = T/N_t$. Let us denote the numerical approximation of the solution by

$$\mathbf{m}_i^n = \mathbf{m}(x_i,t^n) = (u_i^n, v_i^n, w_i^n) \approx \begin{pmatrix} u((i-0.5)h, n\Delta t) \\ v((i-0.5)h, n\Delta t) \\ w((i-0.5)h, n\Delta t) \end{pmatrix}^T,$$

where $i = 1,\dots, N_x$ and $n = 0,1,\dots, N_t$. Let the discrete Laplacian operator be defined as $\Delta_h\mathbf{m}_i = (\mathbf{m}_{i+1} - 2\mathbf{m}_i + \mathbf{m}_{i-1})/h^2$. Then the second-order Crank–Nicolson scheme is given as

$$\frac{\mathbf{m}_i^{n+1} - \mathbf{m}_i^n}{\Delta t} = -\frac{1}{2}\left(\mathbf{m}_i^{n+1}\times\Delta_h\mathbf{m}_i^{n+1} + \mathbf{m}_i^n\times\Delta_h\mathbf{m}_i^n\right) - \frac{\mu}{2}\left[\mathbf{m}_i^{n+1}\times\left(\mathbf{m}_i^{n+1}\times\Delta_h\mathbf{m}_i^{n+1}\right) + \mathbf{m}_i^n \times\left(\mathbf{m}_i^n\times\Delta_h\mathbf{m}_i^n\right)\right]. \qquad (3)$$

We use an accurate and fast nonlinear multigrid method [36,37] for solving the resulting discrete system of Eq. (3). To condense the discussion we describe only the relaxation step in the multigrid method since it is the key step in the algorithm. First, let us rewrite Eq. (3) as

$$\mathbf{m}^{n+1} + \frac{\Delta t}{2}(\mathbf{m}\times\Delta_h\mathbf{m})^{n+1} + \frac{\Delta t}{2}\mu[\mathbf{m}\times(\mathbf{m}\times\Delta_h\mathbf{m})]^{n+1} = \phi^n, \qquad (4)$$

where

$$\phi^n = \mathbf{m}^n - \frac{\Delta t}{2}(\mathbf{m}\times\Delta_h\mathbf{m})^n - \frac{\Delta t}{2}\mu[\mathbf{m}\times(\mathbf{m}\times\Delta_h\mathbf{m})]^n.$$

In its component form, Eq. (4) becomes

$$\begin{pmatrix} u_i^{n+1} \\ v_i^{n+1} \\ w_i^{n+1} \end{pmatrix} + \frac{\Delta t}{2}\begin{pmatrix} v_i\Delta_h w_i - w_i\Delta_h v_i \\ w_i\Delta_h u_i - u_i\Delta_h w_i \\ u_i\Delta_h v_i - v_i\Delta_h u_i \end{pmatrix}^{n+1} + \frac{\Delta t}{2}\mu\begin{pmatrix} v_i(u_i\Delta_h v_i - v_i\Delta_h u_i) - w_i(w_i\Delta_h u_i - u_i\Delta_h w_i) \\ w_i(v_i\Delta_h w_i - w_i\Delta_h v_i) - u_i(u_i\Delta_h v_i - v_i\Delta_h u_i) \\ u_i(w_i\Delta_h u_i - u_i\Delta_h w_i) - v_i(v_i\Delta_h w_i - w_i\Delta_h v_i) \end{pmatrix}^{n+1} = \phi_i^n \qquad (5)$$