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Stability analysis of a parametrically excited functionally graded piezoelectric, MEM system

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ABSTRACT

In this paper the mechanical behavior of a parametrically actuated functionally graded piezoelectric (FGP) clamped–clamped micro-beam is investigated. The micro-beam is supposed to be a composite material with silicon and piezoelectric base. The mechanical properties of the structure, including elasticity modulus, density, and piezoelectricity coefficient are supposed to vary along the height of the micro-beam with an exponential functionality. It is supposed that the FGP clamped–clamped micro-beam is actuated with a combination of direct and alternative electric potential difference. Application of DC and AC actuation voltage leads in a constant and a time-varying axial force. The governing differential equation of the motion is derived using Hamiltonian principle and discretized using expansion theorem with the corresponding shape functions of a clamped–clamped beam. The discretized system is governed by Mathieu equation which's stability is investigated using Floquet theory for single degree of freedom systems and verified using multiple time scales of perturbation technique.

1. Introduction

Nowadays smart materials such as piezoelectric ones are widely being used in design and fabrication processes of MEM and NEM structures. In 2001 Yao et al. analyzed a composite cantilever beam coupling a piezoelectric bimorph to an elastic blade. They analytically investigated the lumped-mass model [1]. In 2002 Takagi et al. designed and fabricated a functionally graded piezoelectric bimorph actuator. They experimentally determined compositional dependency of elastic and piezoelectric properties in the composites; using finite element method they found the optimum compositional profile which led to larger deflection and smaller stress [2]. In 2002 Chen et al. studied on free vibration of a functionally graded piezoelectric rectangular plate. From then on, many researchers have been performed the literature [3]. In 2002 Collet et al. worked on active damping in a micro-cantilever piezocomposite beam; they prepared experimental setups to verify their model. However in many of the published papers in the literature this property of the piezoelectric materials is neglected [4]. In 2005 Zhou et al. simulated and designed a piezoelectric micro-cantilever chemical sensor [5]. Their proposed model consists of an array of multi-layer piezoelectric cantilevers. In 2005 Lu et al. presented the exact solution of a simply supported functionally graded plate laminated under cylindrical bending. To show the influence of material gradients, they gave numerical examples based on the exact solutions [6]. According to a paper by Gao et al. a combination of silicon micro fabrication and piezoelectric thin film deposition is a viable approach to produce miniaturized piezoelectric devices with a complex structure [7]. In 2006 Bian et al. studied on the static and dynamic problems of a simply supported, laminated hybrid FGM beam. The novelty of their work was considering the bonding between the host elastic FGM beam and the piezoelectric layer as a linear spring layer model. They investigated the effects of bonding imperfection on the behavior of the structure [8]. In 2008 Gharib et al. considered a similar model to those of Bian et al. [8] in their model the properties of the FGM layer were functionally graded in the thickness direction. They analytically solved the governing equation and investigated the effects of FGM constituent volume fraction in the peak responses for various volume fraction indexes [9]. In 2009 Rubio et al. designed and modeled FGP

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transducers and compared their achieved results with those of non-FGM ones. They used ANSYS software to analyze their model. The material properties of their proposed model were graded along the thickness direction both linearly and exponentially [10]. In 2009 Xiang et al. [11] considered an FGP micro-beam sandwiched with two piezoelectric layers exposed to electro-thermal load; Their model has been studied in various papers published in the literature including [12–16]. Using Airy stress function they performed static analysis on the proposed model. As a case study they supposed that the material composition varies continuously in the thickness direction according to power distribution law. In 2009 Susanto [17] presented an analytical model for piezoelectric laminated slightly curved beams including computation of natural frequencies, mode shapes and transfer function formulation using distributed transfer function method. In 2010 Simsek [18], analyzed the fundamental frequency of a functionally graded beam having various boundary conditions, using first-order and higher-order shear deformation beam theories; it was supposed that the material properties of the beam vary continuously in the thickness direction, obeying power law distribution. In 2010 Simsek [19] investigated non-linear dynamic analysis of a functionally graded beam with pinned–pinned support due to a moving harmonic load. It was supposed that the material properties of the beam vary continuously along the thickness direction according to a power law form. The non-linear equations of the motion were solved with the aid of Newmark-β method in conjunction with the direct iteration method. In 2010 Alibeigloo [16] presented analytical solution for FGM beams integrated with piezoelectric actuator and sensor under an applied electric field and thermo-mechanical load. The graded properties were assumed to vary exponentially in thickness direction and Poisson's ratio was held constant. In 2010 Simsek [20] investigated the vibration of a functionally graded simply supported beam due to a moving mass using Euler-Bernoulli, Timoshenko and third order shear deformation beam theories. In 2010 Ghazavi et al. performed stability analysis on a piezo-electrically actuated micro-beam. They used Floquet theory for single degree of freedom systems to perform stability analysis [14]. In one of the recently published papers Mohammadi-Alasti et al. [21] studied on the mechanical behavior of a functionally graded cantilever micro-beam subjected to a non-linear electrostatic pressure and temperature variations. It was supposed that the top and bottom surfaces of the micro-beam are made of pure metal and a metal-ceramic mixture respectively. They solved the governing differential equation of the motion using step by step linearization and finite difference methods. Stability analysis is of great importance in the designing process of MEM and NEM devices and has greatly been investigated in the literature [12,14,15,22–25]. In 2011 Azizi et al. [22] Studied on the stabilizing of pull-in instability of a fully clamped piezo-electrically sandwiched micro-beam, subjected to electrostatic actuation. They stabilized the pull-in instability by applying AC voltage with an appropriate amplitude and frequency to the piezoelectric layers; they applied Floquet theory to perform stability analysis on the governing differential equation. As the literature review claims the problems engaged with the dynamics and the statics of the FG and FGP materials are of great interest in these days. One of the mostly concentrated problems in the literature is the parametric excitation and the stability analysis of the microstructures [14,15,22,26–28]. In this paper stability analysis is performed on an FGP micro-beam. The model is supposed to be an FGP clamped–clamped micro-beam in which the material properties including Elasticity modulus, density and the piezoelectric coefficient are graded in the thickness direction according to power law distribution. The FGP micro-beam is exposed to a combination of DC and AC electrical potential difference. The governing differential equation of the motion is derived using Hamiltonian principle and discretized to a one degree of freedom system with a Mathieu type governing ODE using eigen-function expansion theorem. The stability of the governing Mathieu type ODE is investigated using Floquet theory for single degree of freedom systems. To verify the numerically achieved results, the stability boundaries are also obtained using multiple time scale method; the achieved results can be used to stabilize the unstable behavior of the proposed model by applying an AC voltage with an appropriate amplitude and frequency to the structure as proposed in [14,22].

2. Modeling

As illustrated in Fig. 1 the studied model is an FGP clamped–clamped micro-beam of length l, thickness h, and width a. The mechanical properties of the micro-beam are supposed to be graded in the thickness direction of the micro-beam obeying power law distribution. The coordinate system is attached to the mid plane of the left end of the clamped–clamped beam. For a definite z, the mechanical properties are supposed to be a linear combination of the portion of Silicon and that of piezoelectric material. The mechanical properties corresponding to silicon and piezoelectric material are symbolized with subscriptions 'S' and 'P' respectively. As illustrated in Fig. 1, the electrical potential difference is connected to the upper and lower planes of the micro clamped–clamped FGP beam. To create a uniform electric field one needs to have pure conductive metal in the upper and lower planes where the electrical potential difference is connected; to accomplish this, it can be supposed that a thin film metallic layer is etched to the corresponding surfaces without considerably affecting the governing differential equation.

It is considered that the mechanical properties vary with respect to power law distribution as follows:

$$\begin{aligned} E(z) &= E_q\, e^{\gamma|z|} \\ \rho(z) &= \rho_q\, e^{\alpha|z|} \\ e_{31}(z) &= e_{31_p}\, e^{\mu|z|} - \beta \end{aligned} \tag{1}$$

Where E_q, γ, ρ_q, α, μ and β are constants which are determined so that one has:

$$\begin{aligned} z &= 0: \quad MP = MP_0 = P_{s_0}MP_s + P_{p_0}MP_p \\ z &= \frac{h}{2} \rightarrow MP = MP_u = P_{s_u}MP_s + P_{p_u}MP_p \end{aligned} \tag{2}$$

where MP stands for any functionally graded mechanical property including Elasticity modulus, density or piezoelectric coefficient. P_{s_0} and P_{p_0} represent the proportion of silicon and piezoelectric material on mid plane ($z = 0$); Accordingly P_{s_u} and P_{p_u} correspond to those of upper and lower plane $\left(z = \pm\frac{h}{2}\right)$.

Considering the conditions in Eq. (2) the constants will be achieved as follows:

$$\begin{aligned} E(z) &= (E_0)e^{\frac{2}{h}\ln\left(\frac{E_u}{E_o}\right)|z|} \\ \rho(z) &= (\rho_0)e^{\frac{2}{h}\ln\left(\frac{\rho u}{\rho o}\right)|z|} \\ e_{31}(z) &= e_{31_p}\left(e^{\frac{2}{h}|z|\ln\left(1 - P_{p_o} + P_{p_u}\right)} - 1 + P_{p_o}\right) \end{aligned} \tag{3}$$

The governing differential equation of the motion of the micro-beam is derived by the minimization of the Lagrangian. The potential energy includes the following terms [14,22].