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Strong coupling optimization with planar spiral resonators

Avraham Klein, Nadav Katz*

Racah Institute of Physics, Hebrew University of Jerusalem, Edmond J. Safra Campus, Givat Ram, Israel

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ABSTRACT

Planar spirals offer a highly scalable geometry appropriate for wireless power transfer via strongly coupled inductive resonators. We numerically derive a set of geometric scale and material independent coupling terms, and analyze a simple model to identify design considerations for a variety of different materials. We use our model to fabricate integrated planar resonators of handheld sizes, and optimize them to achieve high-Q factors, comparable to much larger systems, and strong coupling over significant distances with approximately constant efficiency.

1. Introduction

Wireless power transfer via resonant magnetic coupling has attracted considerable attention in recent years. This is due both to its elegance and to its possible applicability at many different size scales, from powering spaceships and cars [1,2], and down to handheld scale devices [3] and microdevice coupling [4]. Such coupling depends strongly on two predominantly geometric properties: The devices involved must be high quality resonators, and they must have far-reaching magnetic fields [5,6]. Thus, the geometric design of the resonators is of utmost importance. Moreover, the geometry of a device is inherently size independent, and this means that if a design exists that can be built at different size scales, using different materials, then the same considerations will apply to all variations. The planar spiral is such a design, being both simple and quasi two-dimensional. Thus, for example, planar spiral designs can easily be etched on a thin substrate and incorporated into current handheld devices.

In this Letter, we analyze a planar spiral model and numerically computed coupling terms in order to identify optimal design considerations for different materials at a desired size scale. For example, we show how high T_C superconductors can be designed so as to achieve very strong coupling. We use our method to optimize the design of a device similar in size to currently used handheld devices, using inexpensive materials, by identifying the properties of the dominant dielectric loss channel for the size/materials involved and using capacitative loading to compensate. We achieve very high-Q factors that are typical of much larger devices and strong coupling over significant ranges. We then show that the coupling between the devices is robust, being almost constant over the entire coupling range.

2. Theoretical model

Coupling between two high-Q resonators is adequately described by Coupled-Mode Theory [5,7]. A source and destination device can be represented by complex-valued variables a_1, a_2 normalized so that $|a_n^2|$ is the energy in a resonator, obeying the relation:

$$-i\omega a_1 = -[i\omega_0 + \Gamma_1]a_1 + i\kappa a_2 + F \quad (1)$$

$$-i\omega a_2 = -[i\omega_0 + \Gamma_2]a_2 + i\kappa a_1 \quad (2)$$

where $\Gamma_m = (1+k_m)\gamma_m$ is the loaded dissipation factor of the device, k_m is a coupling coefficient to some load or measuring device and $\gamma_m = \frac{\omega_0}{2Q_m}$ is the unloaded dissipation factor. κ represents the coupling between the two devices and F is a forcing term for the source a_1. Solving Eq. (1)–(2) yields frequency splitting: $|a_2|^2$is maximized and the devices transfer energy efficiently when $\omega - \omega_0 = \pm\sqrt{\kappa^2 - (\Gamma_1^2 + \Gamma_2^2)/2}$. The regime where this splitting takes place is called the strong coupling regime. For identical resonators ($k_1 = k_2 = k_c, \gamma_1 = \gamma_2 = \gamma$) this splitting is possible when:

* Corresponding author. Tel.: +972 2 6586745; fax: +972 2 6586347.
E-mail addresses: avraham.klein@mail.huji.ac.il (A. Klein), katzn@phys.huji.ac.il (N. Katz).

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$$\frac{\kappa}{(1+k_c)\gamma} \geq 1 \quad (3)$$

Thus a Figure of Merit for such a system is the quantity $\kappa/\gamma = Qk$ where k is a dimensionless coupling term. In this we assume $k_c \leq 1$ so as to maintain the strong coupling, or equivalently, keep the loaded quality factor of the devices high. For magnetically coupled systems $k = M/L$, M, L, being the mutual and self inductances of the devices respectively. The efficiency in this regime for identical resonators at the split frequencies is constant and obeys:

$$\eta = \frac{k_c}{2(1+k_c)} \quad (4)$$

The upper limit of $\eta \leq 1/4$ (for $k_c \leq 1$) appears because the devices are identical. Higher efficiency is obtained with proper loading [7,6].

Such coupling can be achieved by using planar spiral resonators driven at quasi-static frequencies. It is well known that such resonators can be modeled as lumped RLC resonant circuits. For a description of the considerations involved in modeling spirals and related planar spirals, we refer the reader to Refs. [8,9]. Fig. 1 shows the schematic of such a setup and a simple equivalent circuit for a spiral. Such a resonator is described by inner and outer diameters d_i, d_o, number of loops n and loop width w, and has a resonant frequency ω_0. Additionally, an underpass strip usually connects the spiral inner and outer extremities, at a separation d_u. Fig. 2 shows a sketch of a basic spiral. Effective L values can be found using current sheet approximations [10]. The capacitance can be written as $C = C_l + C_p$, where C_l, C_p are the inter-coil and coil-underpass capacitances, approximated with coplanar waveguide [11,12](with loops coupling in series) and parallel-plate [13] formulas. The coupling coefficient $k = M/L$ can be extracted numerically. In this work we constructed a table of $k(n, w, d)$ values (d being the coplanar distance between spirals) using the Fasthenry multipole expansion tool [14](see Fig. 3a). These values are scale/material independent so that the same table predicts behavior and design parameters for a wide variety of possible designs. Finally the metallic losses consist of radiative and ohmic losses:

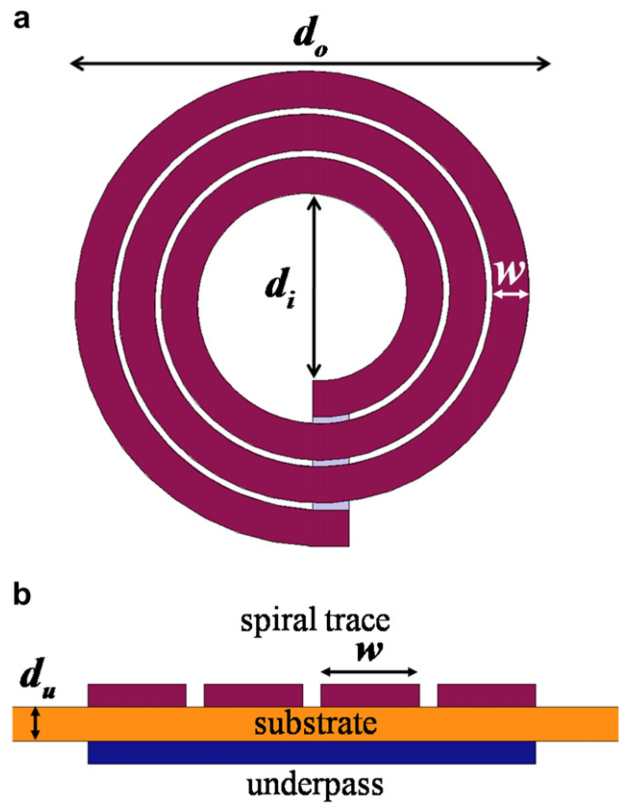

Fig. 2. Sketch of a basic spiral design, and the spiral parameters referred to in our theoretical model. (a) Top-down view, with substrate layer hidden. (b) Side view, with substrate layer.

$$R = \frac{1}{\sigma\xi} \times \frac{2\pi \sum r_i}{2w \cdot \delta_s} + \sqrt{\frac{\mu_0}{\epsilon_0}} \left[\frac{\pi}{6} \left(\frac{\omega}{c} \right)^4 \left(\sum_{i=1}^{n} r_i^2 \right)^2 + \frac{4n^2}{3\pi^3} \left(\frac{\omega}{c} \right)^2 d_u^2 \right] \quad (5)$$

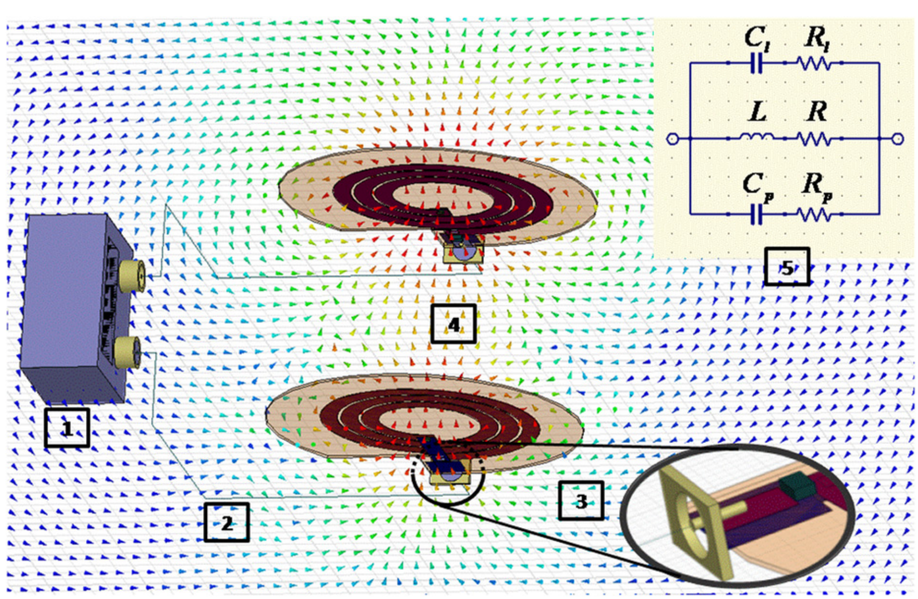

Fig. 1. Schematic of the experimental setup. A network analyzer (1) is connected via coaxial feedlines (2) to a pair of coplanar coupled spirals (the spiral geometry is detailed in Fig. 2). Each spiral is matched to the feedlines via a bonded chip capacitor (3). The S-parameters can be analyzed to obtain the Q factor, coupling and power transfer efficiency between devices. The figure also shows a numerical simulation of the magnetic coupling fields (4), computed using HFSS 11. (5) shows a simplified lumped-element circuit that we have used in our theoretical model.