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The thermal effect on nonlinear oscillations of carbon nanotubes with arbitrary boundary conditions

R. Ansari [a], M. Hemmatnezhad [b,*], J. Rezapour [b]

[a] Department of Mechanical Engineering, University of Guilan, P.O. Box 3756, Rasht, Iran
[b] Faculty of Mechanical Engineering, Islamic Azad University, Lahijan Branch, P.O. Box 1616, Lahijan, Iran

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ABSTRACT

A single-beam model is presented for investigating the nonlinear vibrations of single-walled carbon nanotubes (SWNTs) embedded in an elastic medium. The thermal effect is also incorporated into the formulation. The variational iteration method is used to solve the corresponding nonlinear differential equation. The amplitude–frequency curves for large-amplitude vibrations are graphically illustrated. The influences of thermal effect, some commonly used boundary conditions, changes in material constant of the surrounding elastic medium and variations of geometrical parameters on the vibration characteristics of nanotubes are studied. The results obtained are compared, where possible, with those from the open literature. This comparison clarifies the accuracy as well as the capability of the present method.

1. Introduction

In the last few years, carbon nanotubes (CNTs) have attracted extensive research activities due to their exceptional mechanical, physical, chemical and thermal properties. CNTs were first discovered by Iijima [1] in 1991. A large number of researches have been hitherto conducted to study the mechanical properties of these nanomaterials [2–7]. In spite of being too small and having light weight, they have very large Young's modulus in axial direction (nearly 1 TPa). Undoubtedly, CNTs have the eligibility to be new and the most popular nanomaterial of the 21st century. Since the vibrations of CNTs are of considerable importance in a number of nanomechanical devices such as oscillators, charge detectors, field emission devices and sensors, many researches have been so far devoted to the problem of the vibration of CNTs [8–11]. A good review on the vibration of CNTs is given by Gibson et al. [12] including a concise review of as many of the relevant publications as possible. Based on the theory of thermal elasticity mechanics, Wang et al. [13] studied the vibration and instability analysis of fluid-conveying SWNTs considering the thermal effect.

However, most of the investigations conducted on the vibration of CNTs have been restricted to the linear regime and fewer works were done on the nonlinear vibration of these materials. Recently, Fu et al. [14] studied the nonlinear vibrations of embedded nanotubes using the incremental harmonic balanced method (IHBM). In that work, single-walled nanotubes (SWNTs) and double-walled nanotubes (DWNTs) considered for the study. Ansari et al. [15] applied the homotopy perturbation method (HPM) to investigate the nonlinear vibration of multiwalled carbon nanotubes (MWNTs) using the same model as in [14]. In that paper, they also extended Fu's work to the problem of triple-walled nanotubes (TWNTs) and gave the nonlinear amplitude–frequency curves. In this paper, an elastic Euler–Bernoulli beam model is developed for the nonlinear oscillations of SWNTs taking the thermal effect into consideration. The influences of some commonly used boundary conditions, temperature change and variations of the nanotube's geometrical parameters on the fundamental nonlinear frequencies are examined. The variational iteration method (VIM) is used to formulate solutions to the corresponding nonlinear differential equation.

The aim is to feature the capability of VIM for finding approximate solutions of many nonlinear vibrating systems. The VIM was first proposed as a general Lagrange multiplier method to solve nonlinear problems by Inokuti et al. [16] in 1978. This method has so far been shown to be effective, simple and accurate for solving a large variety of nonlinear problems with approximations converging rapidly to the accurate solutions [17–25]. To illustrate the basic ideas of VIM, consider the following general nonlinear system:

$$Lu(t) + Nu(t) = f(t), \quad (1)$$

where L is a linear operator, N is a nonlinear operator, and $f(t)$ is a known analytic function. According to the variational iteration method, we can construct the following iteration formulation:

* Corresponding author. Tel./fax: +98 141 2229077.
E-mail address: m_hemmatnezhad@yahoo.com (M. Hemmatnezhad).

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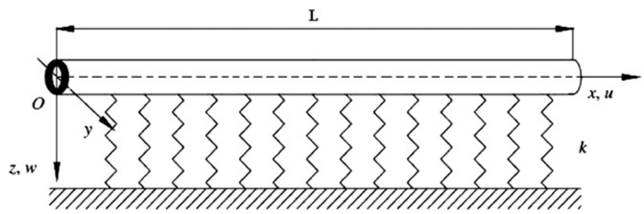

Fig. 1. Model of an embedded carbon nanotube.

$$u_{n+1}(t) = u_n(t) + \int_0^t \lambda\left(Lu_n(\zeta) + N\tilde{u}_n(\zeta) - f(\zeta)\right)\mathrm{d}\zeta. \quad (2)$$

The preceding expression is often called a correction functional whose initial approximation can be taken $u_0(t)$. Here λ is called a general Lagrange multiplier, which can be determined optimally via the variational theory, and $\tilde{u}_n$ is considered as a restricted variation [26], i.e. $\delta\tilde{u}_n = 0$. Now we adopt VIM to the problem of the nonlinear vibrations of CNTs.

2. Basic equations

Consider a CNT of length L, Young's modulus E, density ρ, cross-sectional area A, and cross-sectional inertia moment I, embedded in an elastic medium with constant k determined by the material constants of the surrounding medium. This model is shown in Fig. 1. Assume that u and w are the displacements of the nanotube along x and z directions respectively in terms of the spatial coordinate x and the time variable t.

The free vibration equation of this embedded beam-modeled CNT considering the thermal effect and geometric nonlinearity is [14]

$$\rho A\frac{\partial^2 w}{\partial t^2} + EI\frac{\partial^4 w}{\partial x^4} = \left[\frac{EA}{2L}\int_0^L\left(\left(\frac{\partial v}{\partial x}\right)^2 + \left(\frac{\partial w}{\partial x}\right)^2\right)\mathrm{d}x + N_\mathrm{T}\right]\frac{\partial^2 w}{\partial x^2} + P, \quad (3)$$

where P is the transverse load considered as the interaction pressure per unit axial length between the outermost tube and the surrounding medium, which can be described by the Winkler model [27,28] as

$$P(x,t) = -kw. \quad (4)$$

Here the negative sign indicates that the pressure P is opposite to the deflection of the outermost tube. N_T is the constant axial force associated with the thermal effect defined as [29]

$$N_\mathrm{T} = -\frac{EA}{1-2\nu}\alpha T, \quad (5)$$

where α denotes the coefficient of thermal expansion in the direction of the x-axis, ν is Poisson's ratio, and T is temperature change. Substituting Eqs. (4) and (5) into Eq. (3) gives

$$\rho A\frac{\partial^2 w}{\partial t^2} + EI\frac{\partial^4 w}{\partial x^4} + kw = \left[\frac{EA}{2L}\int_0^L\left(\frac{\partial w}{\partial x}\right)^2\mathrm{d}x + N_\mathrm{T}\right]\frac{\partial^2 w}{\partial x^2}. \quad (6)$$

Assume $w(x,t) = \varphi(x)W(t)$, where $\varphi(x)$ is the first eigenmode of the beam satisfying the kinematic boundary conditions and $W(t)$, is the time-dependent deflection parameter of the nanotube. Applying the Galerkin method, the governing equations of motion are obtained as

$$\frac{\mathrm{d}^2 W}{\mathrm{d}t^2} + \left(\frac{EI}{\rho A}\frac{\alpha_1}{\alpha_2} + \frac{k}{\rho A} - \frac{N_\mathrm{T}}{\rho A}\frac{\alpha_4}{\alpha_2}\right)W - \frac{E}{2\rho L}\frac{\alpha_3}{\alpha_2}W^3 = 0. \quad (7)$$

With the following initial conditions:

$$W(0) = W_\mathrm{max}, \quad \frac{\mathrm{d}W(0)}{\mathrm{d}t} = 0,$$

here W_max denotes the maximum amplitude of oscillation. In Eq. (7) α_1, α_2, α_3 and α_4 are as follows:

$$\begin{aligned}
\alpha_1 &= \int_0^L\left(\frac{\mathrm{d}^4\varphi(x)}{\mathrm{d}x^4}\right)\varphi(x)\mathrm{d}x,\\
\alpha_2 &= \int_0^L\varphi^2(x)\mathrm{d}x,\\
\alpha_3 &= \int_0^L\left[\frac{\mathrm{d}^2\varphi(x)}{\mathrm{d}x^2}\int_0^L\left(\frac{\mathrm{d}\varphi(x)}{\mathrm{d}x}\right)^2\mathrm{d}x\right]\varphi(x)\mathrm{d}x,\\
\alpha_4 &= \int_0^L\left(\frac{\mathrm{d}^2\varphi(x)}{\mathrm{d}x^2}\right)\varphi(x)\mathrm{d}x.
\end{aligned} \quad (8)$$

The deflection of the nanotube is subjected to the following boundary conditions:

For a Simply Supported (S-S) nanotube

$$w(0,t) = 0, \quad \frac{\partial^2 w(0,t)}{\partial x^2} = 0, \quad w(L,t) = 0, \quad \frac{\partial^2 w(L,t)}{\partial x^2} = 0. \quad (9)$$

For a Clamped–Clamped (C–C) nanotube

$$w(0,t) = 0, \quad \frac{\partial w(0,t)}{\partial x} = 0, \quad w(L,t) = 0, \quad \frac{\partial w(L,t)}{\partial x} = 0. \quad (10)$$

For a Clamped–Simply supported (C–S) nanotube

$$w(0,t) = 0, \quad \frac{\partial w(0,t)}{\partial x} = 0, \quad w(L,t) = 0, \quad \frac{\partial^2 w(L,t)}{\partial x^2} = 0. \quad (11)$$

The base functions corresponding to the above boundary conditions are given in Table 1.

3. Application of VIM

Under the transformations $r = \sqrt{I/A}$, and $a = W/r$, Eq. (7) can be transformed to the following nonlinear equation:

$$\frac{\mathrm{d}^2 a}{\mathrm{d}t^2} + f_1 a - f_2 a^3 = 0, \quad (12)$$

with f_1 and f_2 defined as

$$f_1 = \frac{EI}{\rho A}\frac{\alpha_1}{\alpha_2} + \frac{k}{\rho A} - \frac{N_\mathrm{T}}{\rho A}\frac{\alpha_4}{\alpha_2}, \quad f_2 = \frac{EI}{2\rho AL}\frac{\alpha_3}{\alpha_2}. \quad (13)$$

In Eq. (12), $\omega_\mathrm{L} = \sqrt{f_1}$ is the linear, free vibration frequency. Applying VIM constructs the following correction functional on Eq. (12):

$$a_{n+1} = a_n + \int_0^t \lambda(\zeta)\left(\frac{\mathrm{d}^2 a_n(\zeta)}{\mathrm{d}\zeta^2} + f_1 a_n - f_2\tilde{a}_n^3\right)\mathrm{d}\zeta \quad (14)$$

herein $\tilde{a}_n$ is considered as a restricted variation. Making the above correction functional stationary, together with considering $\delta a(0) = 0$, we arrive at

$$\delta a_{n+1}(t) = \delta a_n(t) + \lambda(\delta a_n)'\Big|_0^t - \lambda'\delta a_n\Big|_0^t + \int_0^t(\lambda'' + \lambda f_1)\delta a_n\,\mathrm{d}\zeta = 0. \quad (15)$$