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Semiclassical modelling of cavity quantum electrodynamics with microtoroidal resonators in the weak driving limit

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Abstract

An analytical semiclassical model is presented of the interaction between a single atom and the two degenerate counter-propagating electromagnetic field modes in a microtoroidal resonator. The model is valid in the weak driving limit, and predicts atomic transits with temporal structure qualitatively different than that when only one electromagnetic mode is present.

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Keywords: Cavity quantum electrodynamics; Strong coupling; Semiclassical model; Quantum information

1. Introduction

Cavity quantum electrodynamics facilitates the coherent interaction of single photons with single atoms. Such capabilities are expected to be essential for quantum information networks [5,7], which have the potential to facilitate many novel information processing and communications techniques. Many technology platforms have now been developed [12,16,15,3,13,22], however none presently offer the scalability, efficiency, stability and interaction strength required for a large scale quantum information network. Recently, a new technology based on whispering gallery type microtoroidal optical resonators has been developed [2,20]. Microtoroids have been shown to be highly ideal for cavity quantum electrodynamics [17], and strong coupling between a single Cesium atom and the electromagnetic mode in a microtoroid has recently been demonstrated [1]. Cavity quantum electrodynamics in microtoriods is complicated by the presence of two frequency degenerate counter-propagating electromagnetic field modes coupled by defect scattering [18]. Here, we provide a simple analytic semiclassical model of the system in the weak driving regime, which predicts temporal structure in atomic transits qualitatively different than that present in other systems.

2. Theoretical model of an atom coupled to two electromagnetic modes

In this paper, we model the interaction of a single atom with two modes of the electromagnetic field contained with an optical resonator. A simple model of this system is shown in Fig. 1, with the relevant coupling rates, decay rates, and system operators indicated. In the interaction picture with respect to laser frequency, the Hamiltonian of this system is

$$H = \hbar\Delta_a \hat{b}^\dagger \hat{b} + \hbar\Delta_{c1}\hat{a}_1^\dagger\hat{a}_1 + \hbar\Delta_{c2}\hat{a}_2^\dagger\hat{a}_2 + \hbar g_0\left(\hat{a}_1^\dagger\hat{b} + \hat{b}^\dagger\hat{a}_1\right) + \hbar g_0\left(\hat{a}_2^\dagger\hat{b} + \hat{b}^\dagger\hat{a}_2\right) + \hbar g_c\left(\hat{a}_1^\dagger\hat{a}_2 + \hat{a}_2^\dagger\hat{a}_1\right), \quad (1)$$

where we have used the electric dipole and rotating wave approximations; and $\hat{a}_1$, $\hat{a}_2$ and $\hat{b}$ are annihilation operators describing the electromagnetic field in modes 1 and

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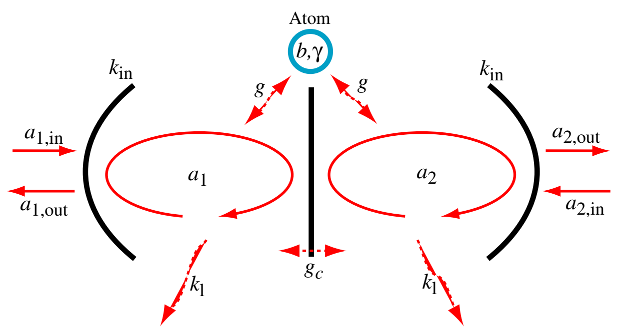

Fig. 1. Schematic of a double resonator symmetrically coupled to a single atom.

2, and the excitation of the atom, respectively; Δ_{c1}, Δ_{c2}, and Δ_a are the electromagnetic and atomic detunings; and g_0 and g_c are the inter-field and field-atom coupling rates with $2g_0$ equal to the well known single-photon Rabi frequency. This expression is a direct extension of the standard Hamiltonian describing a two level atom interacting with one mode of the electromagnetic field given in Doherty [6]. Here, we limit ourselves to the experimentally relevant case of resonant atom and cavity modes, with detuning only for the probe field. Hence $\Delta_{c1} = \Delta_{c2} = \Delta_a = 0$. This simplifies the Hamiltonian in Eq. (1) to

$$H = \hbar g_0\left(\hat{a}_1^\dagger \hat{b} + \hat{b}^\dagger \hat{a}_1\right) + \hbar g_0\left(\hat{a}_2^\dagger \hat{b} + \hat{b}^\dagger \hat{a}_2\right) + \hbar g_c\left(\hat{a}_1^\dagger \hat{a}_2 + \hat{a}_2^\dagger \hat{a}_1\right). \tag{2}$$

Using the quantum Langevin equation this Hamiltonian can be converted into a set of equations of motion for the atomic and intracavity field operators in the usual way [8,9]. Here, we focus on the mean field behavior of the system, and therefore take the expectation values of these equations to yield

$$\dot{\alpha}_1 = -ig\beta - ig_c\alpha_2 - \kappa\alpha_1 - \sqrt{2k_{in}}\alpha_{1,in}, \tag{3}$$

$$\dot{\alpha}_2 = -ig\beta - ig_c\alpha_1 - \kappa\alpha_2, \tag{4}$$

$$\dot{\beta} = -ig\alpha_1 - ig\alpha_2 - \gamma\beta, \tag{5}$$

where the coherent amplitudes $\alpha_1 = \langle\hat{a}_1\rangle$, $\alpha_2 = \langle\hat{a}_2\rangle$, $\beta = \langle\hat{b}\rangle$, and $\alpha_{1,in} = \langle\hat{a}_{1,in}\rangle$, with $\hat{a}_{1,in}$ the annihilation operator for the input probe field; and γ and κ are the atom and cavity decay rates. The cavity decay rate can be expanded as $\kappa = \kappa_{in} + \kappa_l$, where κ_{in} and κ_l are the input coupling and loss rates, respectively. Here, we have neglected terms involving the product of three operators such as $\langle\hat{b}^\dagger\hat{b}\hat{a}\rangle$. This approximation is valid in the weak driving limit of cavity quantum electrodynamics, where the atom is far from saturation. A detailed account of mean field modeling in cavity quantum electrodynamics and a thorough justification of the above approximation can be found in Doherty [6].

In the steady state $\dot{\alpha}_1 = i\omega\alpha_1$, $\dot{\alpha}_2 = i\omega\alpha_2$ and $\dot{\beta} = i\omega\beta$, where ω is the probe detuning. Eqs. (3)–(5) then become a set of simultaneous linear equations which can be solve through application of some simple algebra. The resulting expressions for the intracavity coherent amplitudes α_1 and α_2 are

$$\alpha_1 = \frac{-\sqrt{2\kappa_{in}}\alpha_{in}[\kappa + i\omega + C]}{[\kappa + i\omega + C]^2 - [C + ig_c]^2}, \tag{6}$$

$$\alpha_2 = \frac{\sqrt{2\kappa_{in}}\alpha_{in}[C + ig_c]}{[\kappa + i\omega + C]^2 - [C + ig_c]^2}, \tag{7}$$

where $C = g^2/(\gamma + i\omega)$. We wish to determine the output fields from each cavity mode, which can be obtained using the input/output boundary conditions [8,9]

$$\alpha_{1,out} = \alpha_{in} + \sqrt{2\kappa_{in}}\alpha_1, \alpha_{2,out} = \sqrt{2\kappa_{in}}\alpha_2. \tag{8}$$

We directly find

$$\alpha_{1,out} = \alpha_{in}\left[1 - \frac{2\kappa_{in}[\kappa + i\omega + C]}{[\kappa + i\omega + C]^2 - [C + ig_c]^2}\right], \tag{9}$$

$$\alpha_{2,out} = \frac{2\kappa_{in}\alpha_{in}[C + ig_c]}{[\kappa + i\omega + C]^2 - [C + ig_c]^2}. \tag{10}$$

These equations provide the coherent amplitudes of the output field from the atom – two cavity mode system.

3. Interpretation of model

The response of the system to the probe field predicted from Eqs. (9) and (10) is shown in Fig. 2 as a function of ω for critical coupling and typical values of the coupling rates, and decay rates. The dashed line shows the response of the system when no atom is present ($g = 0$). As expected for a critically coupled cavity [18], when the probe is resonant ($\omega = 0$) the reflected power from the cavity is zero and the transmitted power is maximized, whereas for large detuning ($\omega \gg (\kappa,\gamma)$) this situation reverses. When an atom is present as shown by the solid line, however, one observes Rabi splitting with two spectral resonance appearing sepa-

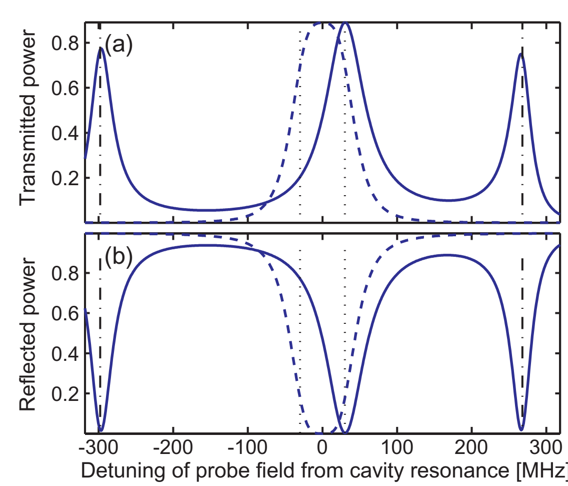

Fig. 2. Cavity (a) transmission and (b) reflection spectra predicted with (solid line) and without (dashed line) the presence of an atom normalized to the probe power. Model parameters: $g_0/2\pi = 200$ MHz, $g_c/2\pi = 30$ MHz, $\gamma/2\pi = 2.6$ MHz, $\kappa_l/2\pi = 1.8$ MHz, critically coupled with $\kappa_{in}/2\pi = 30.05$ MHz.