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Thermo- and soluto-capillary convection in the floating zone process in zero gravity conditions

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Abstract

The study deals with the convective flows in a liquid bridge, maintained between the melting end of a feed rod and the solidifying end of a crystal, in the floating zone process. The surface tension is assumed to be dependent both on the temperature and on the solute concentration. The study is carried out for zero gravity conditions. The free surface deformations and the curvature of the phase change surfaces are neglected. The first part of the study concerns axisymmetric steady flows. Numerical modeling is performed by finite difference method for the parameters, which correspond to the floating zone growth of GeSi alloyed crystal. The calculations show that the evolution of convective flow with the variation of thermal Marangoni number at fixed value of the solutal Marangoni number is accompanied by the hysteresis phenomena, which is related to the existence of two stable steady regimes in certain parameter range. One of these regimes is thermocapillary dominating, corresponding to the two-vortex flow, and the other is solutocapillary dominating, corresponding to the single-vortex flow. The existence of two stable axisymmetric steady regimes in the floating zone process with surface tension depending both on temperature and on the concentration of solute was also observed in Walker et al. [Int. J. Heat Mass Transfer 45 (2002) 4695], where convective flows in the floating zone under strong magnetic field and gravity were studied. The second part of the paper concerns linear stability of axisymmetric steady regimes to three-dimensional perturbations, periodical in azimuthal direction. Two methods are applied to study the stability. First method is based on the direct numerical simulation of temporal evolution of small perturbations of basic state. According to the second method, the exponential dependence of the perturbations on time is assumed. The discretization of the equations by finite difference method leads to the generalized eigenvalue problem. Numerical solution of this problem allows to determine the boundaries of the stability of axisymmetric steady regimes to three-dimensional perturbations with different azimuthal numbers. Stability maps in the parameter plane thermal Marangoni number–solutal Marangoni number are obtained for different values of crystallization velocity and aspect ratio.

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1. Introduction

Let us consider convective flows in a liquid bridge maintained between the melting end of a feed rod and the solidifying end of a crystal in the floating zone process in zero gravity conditions. The surface tension is assumed to be dependent both on the temperature and on the solute concentration. The free surface deformations and the curvature of the phase change surfaces are neglected. We obtain the following dimensionless momentum, continuity, energy and concentration equations:

$$\frac{\partial \vec{V}}{\partial t} + \left(\left(\vec{V} - V_g\vec{e}\right)\nabla\right)\vec{V} = -\nabla p + \Delta\vec{V}, \quad (1)$$

$$\nabla\vec{V} = 0, \quad (2)$$

$$\frac{\partial C}{\partial t} + \left(\left(\vec{V} - V_g\vec{e}\right)\nabla\right)C = \frac{1}{Sc}\Delta C, \quad (3)$$

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$$\frac{\partial T}{\partial t}+\left(\left(\vec{V}-V_{\mathrm{g}}\vec{e}\right)\nabla\right)T=\frac{1}{Pr}\Delta T. \quad (4)$$

The boundary conditions are:
at the liquid–solid interfaces:

$$\text{at } z=0: \quad \vec{V}=0, \quad T=0,$$
$$\left(\frac{\partial C}{\partial z}\right)_{\mathrm{S}}=-Sc\,V_{\mathrm{g}}(1-k_0)C|_{\mathrm{S}}, \quad (5)$$

$$\text{at } z=L: \quad \vec{V}=0, \quad T=0,$$
$$\left(\frac{\partial C}{\partial z}\right)_{\mathrm{S}}=-Sc\,V_{\mathrm{g}}(C-1)|_{\mathrm{S}}, \quad (6)$$

at the free surface $r=1$:

$$V_r=0, \quad (7)$$

$$\frac{\partial V_z}{\partial r}=-Ma_{\mathrm{T}}\,Pr^{-1}\left(\frac{\partial T}{\partial z}\right)+Ma_{\mathrm{C}}\,Sc^{-1}\left(\frac{\partial C}{\partial z}\right), \quad (8)$$

$$\frac{\partial V_\varphi}{\partial r}=V_\varphi-Ma_{\mathrm{T}}\,Pr^{-1}\left(\frac{\partial T}{\partial \varphi}\right)+Ma_{\mathrm{C}}Sc^{-1}\left(\frac{\partial C}{\partial \varphi}\right), \quad (9)$$

$$\frac{\partial C}{\partial r}=0, \quad (10)$$

$$\frac{\partial T}{\partial r}=-Bi(T-T_a). \quad (11)$$

Here k_0 is the segregation coefficient, $T_a=\mathrm{e}^{-(2z-L)^2}$.

Equations and boundary conditions are written in dimensionless form. The following quantities are used as the scales for the velocity, pressure, solute concentration, temperature, time and length: $[V]=\nu/R$, $[p]=\rho\nu^2/R^2$, $[C]=C_{\mathrm{f}}$, $[T]=\Delta T$ ($\Delta T=T_1-T_0$), $[t]=R^2/\nu$, $[r]=R$. Dimensionless parameters of the problem are Prandtl number $Pr=\nu/\chi$, Schmidt number $Sc=\nu/D$, dimensionless crystal growth rate $V_{\mathrm{g}}=u_{\mathrm{cr}}R/\nu$, Biot number $Bi=\varepsilon\sigma^*T_1^3R/\chi$, thermal Marangoni number $Ma_{\mathrm{T}}=\sigma'_{\mathrm{T}}\Delta TR/(\rho\nu\chi)$, solutal Marangoni number $Ma_{\mathrm{C}}=\sigma'_{\mathrm{C}}C_{\mathrm{f}}R/(D\nu)$ and aspect ratio $L=H/R$, where ν is kinematic viscosity, χ is the thermal diffusivity, ε is effective heat flux coefficient, σ^* is Stefan–Boltzmann constant, C_{f} the initial distribution of solute, ρ is melt density.

Most of the calculations were made for fixed values of the parameters Sc, Pr, Bi, L: $Sc=22.5$, $Pr=0.00771$, $Bi=2.0$, $L=2$. Dimensionless crystallization rate V_{g} was varied in the range from 0 to 0.1 and the values of thermal and solutal Marangoni numbers in the ranges $0\leqslant \mathrm{Ma_T}\leqslant 30-100\,000\leqslant \mathrm{Ma_C}\leqslant 100\,000$.

2. Basic state

The problem allows steady axisymmetric solutions. It is convenient to solve the problem for these solutions in terms of the stream function (ψ) and vorticity (Ω):

$$V_r^0=\frac{1}{r}\frac{\partial\psi}{\partial z}, \quad V_z^0=-\frac{1}{r}\frac{\partial\psi}{\partial r}, \quad \Omega=\frac{\partial V_r^0}{\partial z}-\frac{\partial V_z^0}{\partial r}.$$

Then, the equations and boundary conditions take the following form:

$$\frac{\partial\left(V_r^0\Omega\right)}{\partial r}+\frac{\partial\left(\left(V_z^0-V_{\mathrm{g}}\right)\Omega\right)}{\partial z}=\frac{\partial^2\Omega}{\partial r^2}+\frac{1}{r}\frac{\partial\Omega}{\partial r}+\frac{\partial^2\Omega}{\partial z^2}-\frac{\Omega}{r^2}, \quad (12)$$

$$\frac{\partial^2\psi}{\partial z^2}+\frac{\partial^2\psi}{\partial r^2}-\frac{1}{r}\frac{\partial\psi}{\partial r}-r\Omega=0, \quad (13)$$

$$\frac{\partial\left(V_r^0C^0\right)}{\partial r}+\frac{C^0V_r^0}{r}+\frac{\partial\left(\left(V_z^0-V_{\mathrm{g}}\right)C^0\right)}{\partial z}=\frac{1}{Sc}\left(\partial\frac{\partial^2C^0}{\partial r^2}+\frac{1}{r}\frac{\partial C^0}{\partial r}+\frac{\partial^2C^0}{\partial z^2}\right), \quad (14)$$

$$\frac{\partial\left(V_r^0T^0\right)}{\partial r}+\frac{V_r^0T^0}{r}+\frac{\partial\left(\left(V_z^0-V_{\mathrm{g}}\right)T^0\right)}{\partial z}=\frac{1}{Pr}\left(\frac{\partial^2T^0}{\partial r^2}+\frac{1}{r}\frac{\partial T^0}{\partial r}+\frac{\partial^2T^0}{\partial z^2}\right), \quad (15)$$

$$\text{at } r=0: \quad \psi=\Omega=\frac{\partial C^0}{\partial r}=\frac{\partial T^0}{\partial r}=0, \quad (16)$$

$$\text{at } r=1: \quad \psi=\frac{\partial C^0}{\partial r}=0, \quad \frac{\partial T^0}{\partial r}=-Bi\left(T^0-T_a\right),$$
$$\Omega=Ma_{\mathrm{T}}\,Pr^{-1}\left(\frac{\partial T^0}{\partial z}\right)-Ma_{\mathrm{C}}\,Sc^{-1}\left(\frac{\partial C^0}{\partial z}\right), \quad (17)$$

$$\text{at } z=0: \quad \psi=\frac{\partial\psi}{\partial z}=T^0=0,$$
$$\left(\frac{\partial C^0}{\partial z}\right)_S=-Sc\,V_{\mathrm{g}}(1-k_0)C^0\Big|_S, \quad (18)$$

$$\text{at } z=L: \quad \psi=\frac{\partial\psi}{\partial z}=T^0=0,$$
$$\left(\frac{\partial C^0}{\partial z}\right)_S=-Sc\,V_{\mathrm{g}}\left(C^0-1\right)|_S. \quad (19)$$

Problem (12)–(19) was solved numerically by finite difference method. The results of calculations are presented in Figs. 1 and 2. In the floating zone process, the thermocapillary and solutocapillary mechanisms induce the flows with different structure. The surface tension variations due to the temperature inhomogeneities drive two toroidal axisymmetric cells above and below the hottest center plane of the zone. Solutocapillary flow arising due to the surface tension variations related to the compositional inhomogeneities has one-cellular structure. At small values of the thermal Marangoni number, the contribution of the thermocapillary convection is small and