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Numerical solutions for magnetohydrodynamic flow of nanofluid over a bidirectional non-linear stretching surface with prescribed surface heat flux boundary

B. Mahanthesh [a,b], B.J. Gireesha [b,c], R.S. Reddy Gorla [c], F.M. Abbasi [d], S.A. Shehzad [e,*]

[a] Department of Mathematics, AIMS Institutes, Peenya, 560058 Bangalore, India
[b] Department of Studies and Research in Mathematics, Kuvempu University, Shankaraghatta, 577451 Shimoga, Karnataka, India
[c] Department of Mechanical Engineering, Cleveland State University, Cleveland, OH, USA
[d] Department of Mathematics, Comsats Institute of Information Technology, Islamabad 44000, Pakistan
[e] Department of Mathematics, Comsats Institute of Information Technology, Sahiwal 57000, Pakistan

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A B S T R A C T

Numerical solutions of three-dimensional flow over a non-linear stretching surface are developed in this article. An electrically conducting flow of viscous nanoliquid is considered. Heat transfer phenomenon is accounted under thermal radiation, Joule heating and viscous dissipation effects. We considered the variable heat flux condition at the surface of sheet. The governing mathematical equations are reduced to nonlinear ordinary differential systems through suitable dimensionless variables. A well-known shooting technique is implemented to obtain the results of dimensionless velocities and temperature. The obtained results are plotted for multiple values of pertinent parameters to discuss the salient features of these parameters on fluid velocity and temperature. The expressions of skin-friction coefficient and Nusselt number are computed and analyzed comprehensively through numerical values. A comparison of present results with the previous results in absence of nanoparticle volume fraction, mixed convection and magnetic field is computed and an excellent agreement noticed. We also computed the results for both linear and non-linear stretching sheet cases.

1. Introduction

In the recent world of technology, liquid cooling is the major issue in industrial processes. The liquid cooling methods based on ordinary heat transfer fluid are inefficient and ineffective to fulfil the needs of industrial processes. A new class of nanotechnology has been introduced for the higher cooling efficiency during the industrial manufacturing of products. Nanofluids can be formed by the mixture of solid-liquid which have also capability of enhancing the thermal efficiency of ordinary liquids. The suspension of micrometre and millimetre sized solid particles in ordinary fluids resulted into the models of clogging, sedimentation etc. The nanoparticles are generally made of metals like Au, Ti, Ag, Fe, Cu, Al etc., nitride carbide, and metal oxide CuO, TiO_2 and Al_2O_3. The shape of such particles may be tubular, spherical and rod-like. The production of nanofluids is categorized in two ways namely single step and two steps. The general applications of nanoliquids include generation of new type fuels, industrial and vehicle cooling, reduction of fuel in power generation plant, imaging, sensing, cancer therapy etc. Choi [1] was firstly done the experimental work on nanofluid and deduced that the nanoparticles greatly enhance the thermal conductivity of liquid. After his remarkable work on nanofluid technology, the researchers investigated the mechanism of nanofluids widely which can be noticed in the studies [2–19].

The magneto nanofluids have potential role in the industrial manufacturing and production of biomedical equipments. It is involved in the processes of gastric medications, sterilized devices and wound treatment [20]. The desired effects in industrial applications can be achieved by the use of magnetic field for the manipulation of electrically conducting nanoliquids. Number of diverse investigations has been made in the past to examine the performance of magnetic nanoparticles based suspensions. Nowadays, magnetic nanoparticles based suspensions have been implemented in magnetic resonance imaging, targeted drug release, elimination of tumors and many others. Having all such potential roles of magnetic field in modern industry, Sheikholeslami et al. [19] reported the impact of non-uniform magnetic field in the flow of viscous nanofluid past a lid driven annulus enclosure. Zhang et al. [20] discussed the magneto nanofluid over

* Corresponding author.
E-mail addresses: bmanths@gmail.com (B. Mahanthesh), bjgireesu@rediffmail.com (B.J. Gireesha), r.gorla@csuohio.edu (R.S.R. Gorla), abbasisarkar@gmail.com (F.M. Abbasi), ali_qau70@yahoo.com (S.A. Shehzad).

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a radiative sheet in presence of chemical reaction. They imposed the prescribed heat flux condition at the boundary of surface instead of prescribed surface temperature. Abbasi et al. [21] provided a mathematical model for drug delivery system through the peristaltic transport of magnetic particles in water based liquid. Numerical computations for variable magnetic field nanoliquid problem have been made by Sheikholeslami et al. [22]. They used the two-phase model by considering the effects of thermophoresis and Brownian motion. Melting heat transfer phenomenon of Williamson nanofluid in presence of an applied magnetic field has been numerically explored by Krishnamurthy et al. [23]. Impact of constant magnetic field on three-dimensional flow of rate type fluid in presence of Robin's conditions has been addressed by Shehzad et al. [24]. Khan et al. [25] discussed the nonaligned MHD flow of variable viscosity nanoliquid over a heated surface.

The importance of thermal radiation in modern science and technology is very vital and involved in many engineering and industrial processes like agriculture, electric power, food, medical industry, non-destructive testing, solar cell panels and many others. The thermally radiative surface characteristics and surface thermal radiation properties can be changed and affected seriously by the distribution of depositions on the surfaces. It is very essential to understand the mechanism of thermal radiation to obtain the desired highly quality products in the industrial processes. Mahmoud and Waheed [26] analyzed the effects of variable fluid properties in thermally radiative flow of micropolar fluid by imposing the slip velocity condition. Su et al. [27] utilized the thermal radiation effects in convective flow of viscous fluid induced by a permeable wedge. Combined impacts of thermal-diffusion and diffusion-thermo on the non-Newtonian liquid over a radiative sheet in presence of porous medium have been explored by Mahmoud and Megahed [28]. Das et al. [29] reported the impact of thermally radiative flow in a vertical channel by considering the prescribed surface temperature and heat flux conditions. They presented the exact solutions of velocity and temperature through Laplace transform. Shehzad et al. [30] elaborated the effects of radiation in three-dimensional flow of Jeffrey nanoliquid past a bidirectional stretching surface.

This article is based on the numerical computations for three-dimensional flow of viscous fluid over a nonlinear bidirectional surface. The effects of viscous dissipation, Joule heating, magnetic field and thermal radiation are encountered. We suspended the Cu, Al_2O_3 and TiO_2 types nanoparticles in ordinary based liquid. The governing equations are highly nonlinear and coupled due to consideration of mixed convection. Shooting technique is utilized to find the numerical solutions of governing expressions. A benchmark is made to validate the present methodology and we noticed the good agreement with the previous results in limiting way. The computed results are plotted for multiple values of arising parameters and discussed physically.

2. Mathematical modelling

The physical regime of the present study is demonstrated in Fig. 1, in Cartesian coordinate system. A steady three-dimensional flow of an electrically conducting fluid over a stretching surface in the presence of nanoparticles is considered. Let u, v and w be the velocity components along x, y and z directions correspondingly. The sheet is stretched in two directions (x and y) with the nonlinear velocities $u_w = a(x+y)^n$ and $v_w = b(x+y)^n$ respectively, in where a, b and $n>0$ are constants. A variable kind of magnetic field $B=B_0(x+y)^{\frac{n-1}{2}}$ is applied normal to the z direction. The electric field is neglected. The induced magnetic field is also ignored as of

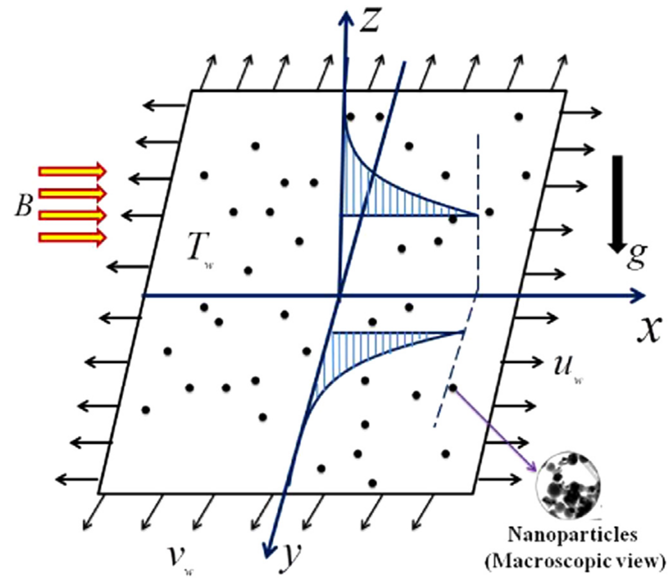

Fig. 1. Geometry of the problem.

small magnetic Reynolds number. Further, the surface is maintained with power law heat flux $q_w=D(x+y)^n$, where D is constant.

We have used the Tiwari and Das model to simulate the nanofluid. The thermophysical properties of the base fluid water and nanoparticles are taken from [5]. It is assumed that, the base fluid and nanoparticles are in thermal equilibrium and no slip occurs between them. The influences of viscous dissipation, Joule heating and thermal radiation are also present in thermal analysis. In accordance with aforementioned assumptions, the conservation of flow and temperature equations are given by

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}=0, \tag{2.1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+w\frac{\partial u}{\partial z} = \frac{\mu_{nf}}{\rho_{nf}}\frac{\partial^2 u}{\partial z^2} + \frac{g(\rho\beta_T)_{nf}}{\rho_{nf}}(T-T_\infty) - \frac{\sigma_{nf}B^2}{\rho_{nf}}u, \tag{2.2}$$

$$u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}+w\frac{\partial v}{\partial z} = \frac{\mu_{nf}}{\rho_{nf}}\frac{\partial^2 v}{\partial z^2} - \frac{\sigma_{nf}B^2}{\rho_{nf}}v, \tag{2.3}$$

$$u\frac{\partial w}{\partial x}+v\frac{\partial w}{\partial y}+w\frac{\partial w}{\partial z} = \frac{\mu_{nf}}{\rho_{nf}}\frac{\partial^2 w}{\partial z^2}, \tag{2.4}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}+w\frac{\partial T}{\partial z} = \frac{k_{nf}}{(\rho C_p)_{nf}}\frac{\partial^2 T}{\partial z^2} + \left[\begin{array}{c}\frac{\mu_{nf}}{(\rho C_p)_{nf}}\left(\left(\frac{\partial u}{\partial z}\right)^2 + \left(\frac{\partial v}{\partial z}\right)^2\right)\\ +\frac{\sigma_{nf}B^2}{(\rho C_p)_{nf}}(u^2+v^2) - \frac{1}{(\rho C_p)_{nf}}\frac{\partial q_r}{\partial z}\end{array}\right], \tag{2.5}$$

the boundary conditions suggested by the physics of the problem