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Influence of convective conditions on three dimensional mixed convective hydromagnetic boundary layer flow of Casson nanofluid

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ABSTRACT

The present work deals with the steady laminar three-dimensional mixed convective magnetohydrodynamic (MHD) boundary layer flow of Casson nanofluid over a bidirectional stretching surface. A uniform magnetic field is applied normal to the flow direction. Similarity variables are implemented to convert the non-linear partial differential equations into ordinary ones. Convective boundary conditions are utilized at surface of the sheet. A numerical technique of Runge–Kutta–Fehlberg (RFK45) is used to obtain the results of velocity, temperature and concentration fields. The physical dimensionless parameters are discussed through tables and graphs.

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1. Introduction

Nowadays researchers are curious to discuss the fluid flow problems over stretching surfaces. Such flows have applications in paper industry, plastic production, textiles, polymer and metal sheets [1–5]. Heat transfer phenomenon with convective boundary conditions is of much more interest due to convection in hot wiring, nuclear plants, frictionless bearing and gas turbine engines etc. Tamayol and Bahrami [6] investigated that the porous materials can be used to enhance heat transfer rate from the stretching surfaces. One can find an admirable literature work in [7–9].

Several biological fluids vary their flow prosperities under the application of applied shear force and therefore show non-Newtonian behavior. One of best fit model among non-Newtonian fluids is Casson fluid model due to its thinning characteristics [10]. Mukhopadhyay [11] mentioned that for different materials Casson model is more useful than viscoplastic model. Casson model has a property of yield stress. Therefore the model can be treated like solid if the yield stress is larger than the shear stress for example drilling operations and metallurgy. If the yield stress is less than the shear stress, the model behaves like liquid and can be applicable to blood, molten chocolate and crude oil. Much useful information can be found in the literature [12,13].

Nanoparticles dispersion in the base fluid known as nanofluids. A contribution of nanoparticles in the base fluid increases the thermal conductivity [14]. Moreover the involvement of less that 1% of nanoparticles in the base fluid almost doubles the thermal conductivity rate [15]. Such enhancement then shows a stable behavior and creates no additional problems like erosion, sedimentation, pressure drop etc. Nanoparticles are useful in nuclear reactors, military systems, electronics, super computers, transformer cooling, biomedicine and extensive range of applications can be found in [16–18]. In recent years energy efficiency is a very busy subject in terms of enhancement in the thermal conductivity. Therefore, the thermal scientists are paying much attention to solar power utilization to find the new energy resources all over the globe. Solar system is considered as one of the best source of energy generation with less amount of environment effect [19–21].

Magnetohydrodynamics (MHD) has key impact in physics, chemistry, industry and engineering. MHD has significance in metal coating, crude oil purification, optical grating, electromagnetic pumps and much more. Furthermore the magneto nanofluid involves both the properties of magnetic and liquid. Nanoparticles show great significance in blood flow analysis, loudspeaker's construction, hyperthermia, kidney transplant etc. Pal et al. [22] solved numerical problem of radiative boundary layer flow of nanofluid passed through a stretching/shrinking sheet embedded in a porous medium using Runge–Kutta–Fehlberg technique. Pal and Mandal [23,24] also studied the above mentioned problem with addition of mixed convection, viscous dissipation and mass transfer effects. Hakeem et al. [25] explored the flow and heat transfer characteristics of MHD stagnation

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point flow of nanofluid over a stretching/shrinking surface using analytical and numerical technique. Hayat et al. [26] presented a mathematical model for the flow problem of three-dimensional flow of Maxwell nanofluid over a sheet with newly introduced zero mass flux nanoparticle condition. The obtained boundary value problem was solved using an analytical technique based on Homotopy Analysis Method (HAM). Shehzad et al. [27] analyzed analytical problem of MHD three-dimensional boundary layer flow of Oldroyd-B nanofluid over a stretching surface with convective boundary conditions. Mahanta and Shaw [28] examined the flow characteristics of MHD three-dimensional flow of Casson fluid over a stretching surface with convective heat condition using Spectral Relaxation Method (SRM). Radiation and chemical reaction effects using convective boundary conditions for problem of three-dimensional boundary layer flow of Casson fluid over a stretching sheet embedded in a porous medium were discussed numerically by Sulochana et al. [29]. Wahiduzzaman et al. [30] analyzed a numerical problem of boundary layer flow of three-dimensional Casson fluid flow over a non-isothermal porous surface. Hayat et al. [31] presented analytical solution of the problem of mixed convective Casson nanofluid with convective conditions over a stretching sheet. Lin et al. [32] investigated the pseudo-plastic nanofluid flow over a finite thin film in presence of heat generation and applied magnetic field. Zhang et al. [33] reported the MHD flow of viscous nanoliquid in a porous medium under variable heat flux and thermal radiation effects.

The current work aims to present a comprehensive numerical study of the problem of three dimensional steady laminar mixed convective MHD flow of Casson nanofluid with radiation, Joule heating, heat source/sink and chemical reaction of first order over a stretchable sheet immersed in a porous medium. Convective boundary conditions on temperature are taken into account. Similarity transformation is utilized to covert non-liner partial differential equations into ordinary ones. The results are obtained using Runge–Kutta–Fehlberg fourth-fifth order (RFK45) technique.

2. Problem formulation

We consider laminar steady three dimensional boundary layer flow of Casson nanofluid over a linear stretching sheet at $z = 0$. The sheet is stretched along x and y-axis with velocity $u = ax$ and $v = by$ respectively and flow takes place in domain $z > 0$. A uniform transverse magnetic field with strength B_o is applied in the normal direction to fluid flow. We assume that the induced magnetic field is negligible and the electric field is neglected [28]. The sheet is heated with the interaction of hot fluid with temperature T_f. The free stream is taken at a constant ambient fluid temperature T_∞ with $T_f > T_\infty$ and concentration C_∞.

The rheological equation of state for an isotropic flow of a Casson fluid [29–30]:

$$\tau_{ij} = \begin{cases} 2\left(\mu_B + p_z/\sqrt{2\pi}\right)e_{ij}, \ \pi > \pi_c \\ 2\left(\mu_B + p_z/\sqrt{2\pi}\right)e_{ij}, \ \pi < \pi_c \end{cases}, \tag{1}$$

where $\pi = e_{ij}e_{ij}$ and e_{ij} stands for the $(i, j)^{th}$ component of deformation rate, π is the product of component of deformation rate with itself, π_c the critical value of this product based on the non-Newtonian fluid, μ_B the plastic dynamic viscosity of the Casson fluid and p_z the yield stress of the fluid.

The fluid flow model in view of above mentioned assumptions is given below:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0, \tag{2}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} = \upsilon\left(1 + \frac{1}{\beta}\right)\left(\frac{\partial^2 u}{\partial z^2}\right) - \frac{\sigma_e {B_0}^2}{\rho_f}u - \frac{\mu\phi_1}{\rho_f k_1}u + g\beta_T(T - T_\infty) + g\beta_C(C - C_\infty), \tag{3}$$

$$u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z} = \upsilon\left(1 + \frac{1}{\beta}\right)\left(\frac{\partial^2 v}{\partial z^2}\right) - \frac{\sigma_e {B_0}^2}{\rho_f}v - \frac{\mu\phi_1}{\rho_f k_1}v, \tag{4}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} + w\frac{\partial T}{\partial z} = \frac{k_o}{\rho_f c_p}\left(\frac{\partial^2 T}{\partial z^2}\right) + \tau D_B\left(\frac{\partial T}{\partial z}\frac{\partial C}{\partial z}\right) + \tau\frac{D_\tau}{T_\infty}\left(\frac{\partial T}{\partial z}\right)^2 + \frac{16}{3}\frac{\sigma {T_\infty}^3}{\rho_f c_p k^*}\left(\frac{\partial^2 T}{\partial z^2}\right) + \frac{\sigma_e {B_0}^2}{\rho_f c_p}(u^2 + v^2) + \frac{Q}{\rho_f c_p}(T - T_\infty), \tag{5}$$

$$u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y} + w\frac{\partial C}{\partial z} = D_B\left(\frac{\partial^2 C}{\partial z^2}\right) + \frac{D_\tau}{T_\infty}\left(\frac{\partial^2 T}{\partial z^2}\right) - K(C - C_\infty), \tag{6}$$

where u, v and w are the velocity components in x, y and z-directions respectively, υ is the kinematic viscosity, β is the Casson parameter, σ_e is the electrical conductivity of the fluid, ρ_f is the density, μ is the viscosity, ϕ_1 is the porosity and k_1 is the permeability of the porous medium, K_0 is the thermal conductivity of the fluid, $\tau = \frac{(\rho c)_p}{(\rho c)_f}$ is the ratio of nano-particle heat capacity and the base fluid heat capacity, D_B is the Brownian diffusion coefficient and D_T is the thermophoretic diffusion coefficient, c_p is the specific heat capacity, σ is the Stephens Boltzmann constant, k^* is the absorption coefficient, Q is uniform volumetric rate of heat absorption/generation and K is the chemical reaction parameter.

The boundary conditions for the velocity components, temperature and concentration are:

$$\begin{cases} u = ax, \ v = by, \ w = 0, \ -k\left(\frac{\partial T}{\partial z}\right) = h_1(T_f - T), \\ \quad -D_B\left(\frac{\partial C}{\partial z}\right) = h_2(C_f - C) \quad at\ z = 0, \\ u \to 0, \ v \to 0, \ T \to T_\infty, \ C \to C_\infty as z \to \infty, \end{cases} \tag{7}$$

where a, $b > 0$ corresponds to the sheet stretching, k_f is the thermal conductivity, h_1 and h_2 are the heat and mass transfer coefficients. To have velocity, temperature and concentration fields, we introduce the following similarity transformations:

$$\begin{cases} u = axf'(\eta), \ v = ayg'(\eta), \ w = -\sqrt{a\upsilon}\,(f(\eta) + g(\eta)), \ \eta \\ \quad = z\sqrt{\frac{a}{\upsilon}}, \\ \theta(\eta) = \frac{T - T_\infty}{T_f - T_\infty}, \ \phi(\eta) = \frac{C - C_\infty}{C_f - C_\infty}, \end{cases} \tag{8}$$

where $c = \frac{b}{a}$ is the velocity ratio along y and x-directions. Using (8) into (2)–(7), we note that the continuity equation is identically satisfied. The (Eqs. (3)–6) take the following dimensionless form:

$$\left(1 + \frac{1}{\beta}\right)f''' - (f')^2 + (f + g)f'' - \left(M^2 + P\right)f' + \alpha(\theta + N\phi) = 0, \tag{9}$$

$$\left(1 + \frac{1}{\beta}\right)g''' - (g')^2 + (f + g)g'' - \left(M^2 + P\right)g' = 0, \tag{10}$$

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