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Review

Hydromagnetic Hiemenz flow of micropolar fluid over a nonlinearly stretching/shrinking sheet: Dual solutions by using Chebyshev Spectral Newton Iterative Scheme

Asad Mahmood [a,*], Bin Chen [b], Abuzar Ghaffari [a]

[a] Department of Mathematics and Statistics, International Islamic University, Islamabad 44000, Pakistan
[b] School of Environment, Beijing Normal University, Beijing 100875, PR China

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ABSTRACT

Hydromagnetic stagnation point flow and heat transfer over a nonlinearly stretching/shrinking surface of micropolar fluid is investigated. The numerical simulation is carried out through Chebyshev Spectral Newton Iterative Scheme, after transforming the governing equations into dimensionless boundary layer form. The dual solutions are reported for different values of magnetic and material parameters against the limited range of stretching/shrinking parameter. It is also noted that second solution only occurs for the negative values of stretching/shrinking parameter, whereas for the positive values unique solution exists. The effects of dimensionless parameters are described through graphs. It is seen that the flow and heat transfer rates can be controlled through the material parameter and magnetic force.

1. Introduction

In recent years, the study of fluid flow and heat transfer phenomenon has got much attention due to their vast applications in industries and other technical areas. Flow due to stretching/shrinking sheet has much importance in metal and polymer processing, such as wire drawing, wire coating, cooling of infinite metallic plates, paper production, drawing of plastic film and rubber sheets, etc. The quality of production is much decisively influenced by the kinematic of stretching and simultaneous heating or cooling, during the process. The classical two-dimensional flow was first analyzed by Hiemenz [1] and then Homann [2] extended it to an axisymmetric case. The analytical solution for the boundary layer flow of an incompressible viscous fluid over a stretching and shrinking sheet was first found by Crane [3] and Wang [4]. Due to contribution of Crane [3] and Wang [4] many researchers extended the study of fluid flow over a stretching/shrinking sheet under the action of different physical conditions. According to literature survey recently, flow over shrinking surface has received much attention of the researchers as compared to stretching surface, due to its interesting behavior.

In fluid dynamics, the study of stagnation point flow has gain much attention due to its vast industrial applications such as cooling of nuclear reactors, cooling of electric devices by fans, solar central receivers, which are exposed to wind currents and many hydrodynamics processes in engineering, etc. Recently, the study of stagnation-point flow of micropolar fluid has attracted many researchers. The micropolar fluid dynamics is concerned with the motion of the fluid whose material point possess orientations and it is different from the classical fluid dynamics. In classical fluid dynamics it is assumed that the material point does not possess the orientations. Micropolar fluid was first introduced by Eringen [5,6], which is capable of analyzing those fluids having microscopic effects and micromotion. Many researchers investigated the flow of micropolar fluid under different aspects, some of them are Nazar et al. [7], Lok et al. [8], Ishak et al. [9], Hayat et al. [10] and Yacob and Ishak [11].

The study of mutual interaction between moving fluid and magnetic field is commonly known as (magnetohydrodynamics) MHD. The study of MHD has received the intention of researchers and scientists due to its applications in industrial technology such as liquid metal flow control, MHD power generators, micro MHD pumps, drying processes and solidification of binary alloy, etc. MHD effects the micropolar fluid because it is a very good conductor and exhibits very low dissipative effect [12]. When we apply the Lorentz force normal to the fluid motion, it has impact on charged particles, which definitely effects the fluid motion. The influence of external magnetic field on two dimensional stagnation point flow towards a shrinking sheet was analyzed by Lok et al. [13] and Ishak et al. [14] in which effects of stretching/

* Corresponding author.
E-mail address: asadmahmood_86@yahoo.com (A. Mahmood).

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shrinking parameter and magnetic parameter near stagnation point are investigated and heat transfer phenomenon is discussed. The analytical solution of boundary layer flow of a micropolar fluid towards a stretching sheet with MHD was found by Hayat et al. [10]. MHD boundary layer flow and heat transfer of an electrically conducting micropolar fluid over a nonlinear stretching surface with variable wall heat flux in the presence of heat generation/absorption was investigated by Mahmoud and Waheed [15]. Recently, Sheikholeslami and Ganji [16] studied the effect of external magnetic field to investigate ferrofluid behavior. They used numerical simulation to obtain the results for Nusselt number with the variation of emerging parameters. In another article Sheikholeslami [17] has investigated the magnetic effect on ferrofluid in an enclosure, where the internal cylindrical surface is heated with uniform flux. The magnetic effect on the nanofluid between two parallel plates which are rotating with angular velocity is carried out by Sheikholeslami et al. [18]. MHD effect on time dependent flow of Newtonian fluid in stagnation point region, where the fluid is impinging obliquely to the surface was considered by Javed et al. [19]. Very recently, the studies on the topic of uniform and non-uniform hydromagnetic flows can be seen in Refs. [20–28].

The present paper is concerned to investigate the numerical solution of hydromagnetic Hiemenz flow and heat transfer of micropolar fluid over a impermeable nonlinearly stretching/shrinking sheet, which is not considered in the previous studies. The similarity solution of obtained system of equations is achieved numerically through CSNIS [33,34]. There are many other numerical techniques (see Refs. [29–32]) that can be used to tackle such problems but the used numerical scheme has some advantages over the few other numerical techniques. In current technique, we transform the domain of given problem to $[-1, 1]$. Further, we discretize this domain into few number of grid points, while using some other numerical methods such as shooting, finite difference and finite element, etc. we discretize the domain in thousands of grid points to get accuracy. This method can also predict the dual solutions very easily as compared to other numerical techniques. For validity of numerical results, we compared our results with previously published studies (see Table 1). To check the convergence speed, the values of $g''(0)$ for different iterations are given in Table 2. It is observed that the numerical scheme employed in the present problem is efficient, less time consuming, stable and rapid convergent.

2. Mathematical formulation

We considered steady two-dimensional hydromagnetic Hiemenz flow of an incompressible and electrically conducting micropolar fluid towards a impermeable nonlinearly stretching/shrinking sheet. The magnetic field of strength B_0 is applied normal to the direction of fluid motion. The induced magnetic field is neglected due to low magnetic Reynolds number assumption. Also the joule heating effect is neglected due to small magnetic interaction parameter because this effect becomes more important for the sufficiently strong applied field. We also considered $U_e(x) = ax^n$, $U_w(x) = bx^n$ and $T_w(x) = T_\infty + cx^{2n}$ as free stream velocity, stretching/shrinking velocity and surface temperature respectively, which are varying nonlinearly. Where $a > 0$ and $b, c, n \geq 0$ are constants and T_∞ is the temperature far from the surface. Therefore the governing equations of the flow and heat transfer are given as (see Refs. [9–11]).

Table 1
Comparison of $g''(0)$ for different values of ε and K, when $Pr = 1$, $M = 0$ and $n=1$.

K	ε	Zaimi et al. [40]		Present (using CSNIS)	
		First sol.	Second sol.	First sol.	Second sol.
0	0.5	0.713294		0.713295	
	−1.2	0.932473	0.233649	0.932474	0.233646
0.1	0.5	0.696104		0.696105	
	−1.2	0.910000	0.228018	0.910001	0.228014
0.5	0.5	0.637990		0.637990	
	−1.2	0.834029	0.208982	0.834030	0.208978
1	0.5	0.582402		0.582403	
	−1.2	0.761361	0.190771	0.761361	0.190765

Table 2
Values of skin friction coefficient $g''(0)$ at different iterations when $\varepsilon = 0.5$.

Iterations	$K=0$		$K=1$	
↓	$M=0$	$M=1$	$M=0$	$M=1$
1	3.987270	1.126375	3.516463	0.954794
2	2.102631	0.891217	1.820610	0.732711
3	1.232995	0.870217	1.040969	0.710706
4	0.869397	0.869626	0.717056	0.710047
5	0.744089	0.869624	0.607548	0.710045
6	0.715699	0.869624	0.584131	0.710045
7	0.713318	0.869624	0.582415	0.710045
8	0.713295	0.869624	0.582403	0.710045
Zaimi et al. [40] ⟶	0.713294	–	0.582402	–

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = U_e\frac{dU_e}{dx} + \frac{\mu + \kappa}{\rho}\frac{\partial^2 u}{\partial y^2} + \frac{\kappa}{\rho}\frac{\partial N}{\partial y} + \frac{\sigma B^2(x)}{\rho}(U_e - u) \tag{2}$$

$$\rho j\left(u\frac{\partial N}{\partial x} + v\frac{\partial N}{\partial y}\right) = \gamma\frac{\partial^2 N}{\partial y^2} - \kappa\left(2N + \frac{\partial u}{\partial y}\right) \tag{3}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{k}{\rho c_p}\frac{\partial^2 T}{\partial y^2} \tag{4}$$

where u and v are the components of velocity in the x- and y-directions respectively, N is the component of the micro-rotation vector normal to the xy-plane, T is the fluid temperature, σ is the electrical conductivity of fluid, $B^2(x) = B_0^2 x^{n-1}$ is the electromagnetic induction (Shen et al. [39]) , k is the thermal conductivity, γ is the spin gradient viscosity, κ is the microrotation viscosity, c_p is the specific heat at constant pressure and ρ is the density of fluid.

Eqs. (1)–(4) are subjected to boundary conditions as

$$u = U_w, \quad v = 0, \quad N = -m\frac{\partial u}{\partial y}, \quad T = T_w \text{ at } y = 0$$
$$u = U_e, \quad N = 0, \quad T = T_\infty \text{ as } y \to \infty \tag{5}$$

where m is constant, whose value is $0 \leq m \leq 1$ and for $m = \frac{1}{2}$ antisymmetric part of stress tensor cancels (Ahmadi [35]). Consider $\gamma = \left(\mu + \frac{\kappa}{2}\right)j = \mu\left(1 + \frac{K}{2}\right)j$ (Ishak et al. [9], Ahmadi [35]), where $K = \frac{\kappa}{\mu}$ is the material parameter.

We use the following transformations to obtain the similarity solution of Eqs. (1)–(4), with boundary conditions (Eq. (5)), see Ishak et al. [9]