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Mode coupling in spin torque oscillators

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ABSTRACT

A number of recent experimental works have shown that the dynamics of a single spin torque oscillator can exhibit complex behavior that stems from interactions between two or more modes of the oscillator, such as observed mode-hopping or mode coexistence. There has been some initial work indicating how the theory for a single-mode (macro-spin) spin torque oscillator should be generalized to include several modes and the interactions between them. In the present work, we rigorously derive such a theory starting with the Landau–Lifshitz–Gilbert equation for magnetization dynamics by expanding up to third-order terms in deviation from equilibrium. Our results show how a linear mode coupling, which is necessary for observed mode-hopping to occur, arises through coupling to a magnon bath. The acquired temperature dependence of this coupling implies that the manifold of orbits and fixed points may shift with temperature.

1. Introduction

Since the prediction of spin transfer torque (STT) in 1996 [1–3], whereby a spin-polarized dc current exerts a torque on the local magnetization order parameter, there has been a wealth of theoretical and experimental investigations of phenomena driven by STT. One particular manifestation of STT is the spin torque oscillator (STO). The STO is typically realized in MgO magnetic tunnel junctions [4–8], or metallic nanocontacts [9–11]; in both of these, a dc current is driven perpendicularly to two thin stacked magnetic layers, in one of which the magnetization is relatively free to rotate, while in the other the magnetization is held fixed. With the relative magnetization directions and current direction arranged appropriately, STT pumps energy into the STO, and by adjusting the current magnitude, this pumping can be made to cancel the intrinsic dissipative processes in the system. This gives rise to almost undamped oscillations with a very small linewidth. As STOs are potentially useful in technological applications, such as frequency generators or modulators, it is both of practical as well as of fundamental interest to understand the physics of the STO auto-oscillations. Slavin and co-workers [12–15] put forth a comprehensive theory valid for single-mode STOs, that is, STOs for which one mode is relevant and is excited (this is when a macro-spin model is readily applicable). Some striking features of this theory are the effects induced by the inherent nonlinearity of the STOs, for example the behavior of the oscillator linewidth below and above threshold current [13–15] at which STT pumping first cancels damping and auto-oscillations are achieved.

Recently, there have been several experiments demonstrating the effects of multi-mode STOs, for example mode co-existence and mode-hopping [16–23]. Clearly, the interactions between several oscillator modes cannot be described by the single-mode theory but require a theory that describes the interactions between collective modes, and how the behavior of the collective modes is modified as a consequence of those interactions. de Aguiar et al. [24] considered a two-mode system with in-plane translational invariance and arrived at coupled equations similar to those of the Lotka–Volterra biological model. However, de Aguiar et al. argued that in the driven two-mode system, one mode will get extinguished and only one mode survives. This is obviously not consistent with many experimental observations of mode-hopping and mode-coexistence. A multi-mode theory was first outlined by Muduli et al. [22,8,25]. In particular, these authors argued that the equations describing two coupled modes could be

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mapped onto a driven dynamical system used to describe semiconductor ring lasers [26,27]. It is known that in the presence of thermal noise, those equations exhibit mode-hopping in certain regions of parameter space [27]. A key observation here was that for mode-hopping to be present in a two-mode system, the time derivative of the slowly varying amplitude of one mode must be coupled linearly to the amplitude of the other mode (a so-called "backscattering" term). Also, the authors gave some general argument for why mode-hopping is a minimum when the free layer magnetization is anti-parallel to that of the fixed layer, and then increases as the orientation moves away from anti-parallel [22]. Later, Iacocca et al. [28] also showed that the effective multi-mode theory described very well observed line-width broadening near mode crossings [8], and Sharma et al. [29] observed a $1/f$-type frequency noise spectrum in magnetic tunnel junction STOs, which was attributed to mode hopping. Iacocca et al. [30] also presented experimental results that identified two mode coupling mechanisms: magnon-mediated scattering, and intermode interactions consistent with preliminary results of the present work [31].

With the mounting experimental observations that appear to be consistent with the multi-mode theory of Muduli et al. [22], it is of interest to present a rigorous derivation of the equations for coupled modes from first principles (the micromagnetic Landau–Lifshitz–Gilbert equation), and to analyze the ensuing behavior of the system. That is one purpose of the present work. We will show how the linear backscattering term arises naturally in a system with a small number, e.g., two, of dominant modes but in which there is a bath of many modes. This bath provides effective interactions between the dominant modes when the bath is integrated out and the equations projected onto the subspace of dominant modes. We will also derive and discuss some consequences of this theory. The backscattering terms have a direct temperature dependence as they involve thermal populations of modes. This has a consequence of a temperature dependence of the magnitude of the backscattering terms, as we will explicitly show in an example, which leads to a temperature dependence of the linewidths of the modes [28]. The temperature dependence of the backscattering terms also has a more subtle consequence in that manifold of orbits and fixed points will shift as a function of temperature, which will change the location or height of saddle points that are crossed during mode-hopping.

While the algebra may at times seem a bit tedious, we are writing out some of the expressions explicitly to point out symmetries and physical consequences. We will also present some examples to illustrate how mode hopping can arise from mode interactions. Our results will show that there is always some small possibility of mode hopping, consistent with experimental observations [22]. However, the backscattering term that leads to mode hopping grows with the appearance of nutation in the modes of the system, in which the phase between the dynamical components of the magnetization motion is not constant across the magnetic layer. For a system such as the MTJ with an in-plane magnetic field studied in Ref. [22], we will show how the backscattering increases as the external field is rotated away from the direction of the fixed layer, consistent with experimental observations [22]. Finally, we note that the method we use here is based on an expansion in eigenmodes of the linearized conservative equations of motion for the magnetization and will therefore not apply to, e.g., systems that exhibit a localized bullet mode [32].

2. Methods

2.1. Micromagnetic equations

Our starting point is a soft ferromagnetic system, for example a thin film. We describe the local magnetization by a director $\hat{m}_i$ for discrete sites $i = 1, 2, \ldots N$, with $|\hat{m}_i| = 1$. The LLG equation including damping and spin torque is then

$$\frac{d\hat{m}_i}{dt} = -\gamma\hat{m}_i \times \mathbf{H}_{\mathrm{eff},i} - \frac{\gamma\alpha}{1+\alpha^2}\hat{m}_i \times \left[\hat{m}_i \times \mathbf{H}_{\mathrm{eff},i}\right] + \gamma a_J \hat{m}_i \times \left[\hat{m}_i \times \hat{M}\right]. \quad (1)$$

Here, γ is the gyromagnetic ratio, $\alpha \ll 1$ the dimensionless damping, a_J the effective field due to STT, and $\hat{M}$ the (uniform) magnetization direction of the fixed layer; the effective field $\mathbf{H}_{\mathrm{eff},i}$ includes exchange, demagnetizing fields, and an external applied field $\mathbf{H}_{\mathrm{ext}} = H_{\mathrm{ext},x}\,\hat{x} + H_{\mathrm{ext},y}\,\hat{y} + H_{\mathrm{ext},z}\,\hat{z}$. We will not here include Oersted fields generated by the currents in the system as they are not important for the present analysis, although it has been shown that these fields play an important role in the interactions between certain modes in nano-contact STOs [23]. We are also ignoring the so-called field-like, or perpendicular, spin torque [33] as this can be absorbed into the definition of the external field. We shall combine exchange and demagnetizing fields into a single field $\mathbf{H}_{d,i}$ and note that in general we can write

$$H_{d,i,\delta} = \sum_{i',\epsilon} D_{i,i';\delta,\epsilon} m_{i',\epsilon}, \qquad \delta, \epsilon = x, y, z, \quad (2)$$

where $D_{i,i';\delta,\epsilon}$ is a generalized demagnetizing tensor that includes near-neighbor exchange. We shall also assume that the magnetocrystalline anisotropy is negligible, and we take the $\hat{x}$-axis to be along the average equilibrium magnetization in the free layer. Without loss of generality, we can take the $\hat{y}$-axis also to be in the $\hat{x}'\hat{y}'$-plane. Fig. 1 illustrates the geometry of the system. In general, the demagnetizing tensor will have all off-diagonal terms nonzero in the representation of the xyz coordinate system. However, for some specific examples such as magnetic tunnel junctions with an in-plane external field, or systems with a large $(H_{\mathrm{ext}} \gg 4\pi M_S)$ external field perpendicular to the planes of the magnetic layers, the demagnetizing tensor can be taken to be (approximately) diagonal in the xyz coordinate system. In such cases, if the external field is in the $\hat{x}'\hat{z}'$-plane, the only nonzero off-diagonal elements are $D_{i,i';x,z} = D_{i,i';z,x}$. Fig. 2 depicts the equilibrium magnetization in the free layer of a circular magnetic tunnel junction STO of diameter $d = 240$ nm obtained from micromagnetic simulations with parameters appropriate for the systems in Ref. [34]. In the figure, the pinned layer and reference layer magnetizations are approximately (these layers are also treated micromagnetically) at 45° and −135° degrees to the x-axis, and there is an external field of magnitude 450 Oe applied in the xy-plane at 85° to the x-axis (in an actual magnetic tunnel junction, there are three magnetic

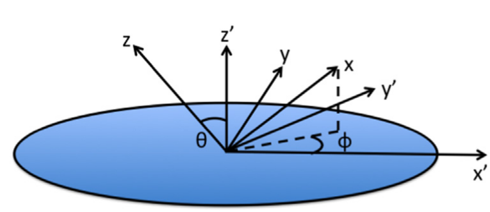

Fig. 1. Geometry used in this work. The FL is in the $\hat{x}'\hat{y}'$-plane; the $\hat{x}$-axis is along the FL magnetization equilibrium direction, which may point out of the $\hat{x}'\hat{y}'$-plane because of the applied field $\mathbf{H}_{\mathrm{ext}}$. The projection of the $\hat{x}$-axis on the $\hat{x}'\hat{y}'$-plane is rotated an angle ϕ from the $\hat{x}'$-axis. The $\hat{y}$-axis is in the $\hat{x}'\hat{y}'$-plane, and the $\hat{z}$-axis is rotated an angle θ from the $\hat{z}'$-axis.