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Magnetic field analysis in a suspension of gyrotactic microorganisms and nanoparticles over a stretching surface

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ABSTRACT

The combine effects of magnetic field bioconvection, Brownian motion and thermophoresis on a free convection nanofluid flow over a stretching sheet containing gyrotactic microorganisms are investigated. The self-similar Buongiorno model is analyzed first time for stretching sheet numerically. The present results are compared with available data and are found in an excellent agreement. Pertinent results are presented graphically and discussed quantitatively with respect to variation in bioconvection parameters.

1. Introduction

The term bioconvection refers to macroscopic convection persuaded in aquatic by the communal motion of a large number of self-propelled motile microorganisms [1]. Like natural convection, bioconvection is caused by unbalanced thickness stratification, and it is frequently categorized by regular fluid flow patterns. Linear solidity enquiry of a postponement of gyrotactic microorganisms in an isothermal fluid layer of finite depth is carried out in Hill et al [2]. There is important impending in applications of nanofluids in numerous sorts of microsystems. These comprise micro-heat cylinders [3], microchannel heat sinks [4], and microreactors [5]. There is also a sturdy interest in using nanomaterials in dissimilar bio-microsystems, such as enzyme biosensors [6]. Munir et al. [7] give the dynamics of capturing process of multiple magnetic nanoparticles in a flow through microfluidic bioseparation system. There is also interest in developing chip-size microdevices for evaluating nanoparticle toxicity. Huh et al. [8] suggested a reconstituting organ-level lung functions on a chip. Bioconvection has latent requests in bio-microsystems due to mass transport improvement and mixing, which are vital issues in many microsystems [9,10]. Also, the consequences obtainable in Shitanda [11] propose using bioconvection in a toxic multiple sensor due to the aptitude of some toxic compounds to oblige the flagella movement and thus overpower bioconvection.

The two-dimensional flow analysis towards a linear stretching sheet attained great interest because of its constituent application in industrial and engineering area for growing and constricting of exteriors such as shrinking/stretching covering, bundle packaging, hot undulating, extrusion of area substantial, glass fiber, rope rolling, metallic packaging and aluminum jug industrialized developments. Primarily, Crane [12] extended the idea of stretching sheet and he found the closed form solution for the viscous fluid over a stretching surface. Idea of boundary layer flow towards a poignant surface was examined by Sakiadis [13]. Since then the idea of stretching sheet for both Newtonian and non-Newtonian fluid has been extensively studied [14–17].

In recent times, nanofluids have accomplished marvelous consideration due to its useful applications. Nanofluids are in fact homogeneous combination of base fluid and nanoparticles. The idea of nanofluids raises to a novel caring of heat transport fluids by appending nano-scaled particles metals and nonmetals in base fluids. At first, the name of nanofluid was offered by Choi [18], in which he defined the interruption covering nanoparticles of diameter less than 50 nm. After Choi [18], many researchers present the nanofluid model analysis for various fluids see Refs. [19–28].

The aim of the present paper is to discuss the boundary layer mixed convection flow over a stretching sheet with a water-based nanofluid containing gyrotactic microorganisms. The similarity transformations are incorporated in the analysis to simplify the problem. The influence of bioconvection parameters on the

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dimensionless velocity, temperature, nanoparticle concentration and density of motile microorganisms as well as on the Skin friction coefficients, local Nusselt, Sherwood and motile microorganism numbers are investigated and presented graphically.

2. Formulation of the problem

Consider two-dimensional steady incompressible fluid past a stretching sheet. In addition, MHD and nanoparticle effects are saturated with gyrotactic microorganisms, while sheet is stretching with the plane $y=0$. The flow is assumed to be confined to $y>0$. Here we assumed that the sheet is stretched with the linear velocity $u(x)=ax$, where $a>0$ is constant and the x-axis is measured along the stretching surface. A uniform constant magnetic field is applied normal to stretching surface. Whereas, the effects of induced magnetic field are negligible.

$$\bar{\nabla}\cdot\bar{V}=0, \tag{1}$$

$$\rho_f\left(\frac{\partial\bar{V}}{\partial t}+\bar{V}\cdot\bar{\nabla}\bar{V}\right)=-\bar{\nabla}p+\mu\bar{\nabla}^2\bar{V}+J\times B+[\bar{C}\rho_p+\left(1-\bar{C}\right)\{\rho_f[1-\beta(\bar{T}-\bar{T}_\infty)]\}\}+\left(\bar{n}-\bar{n}_\infty\right)]g, \tag{2}$$

$$(\rho c)_f\left(\frac{\partial\bar{T}}{\partial t}+\bar{V}\cdot\bar{\nabla}\bar{T}\right)=k\bar{\nabla}^2\bar{T}+(\rho c)_f\left[D_B\bar{\nabla}\bar{C}\cdot\bar{\nabla}\bar{T}+\left(\frac{D_T}{\bar{T}_\infty}\right)\bar{\nabla}\bar{T}\cdot\bar{\nabla}\bar{T}\right], \tag{3}$$

$$\left(\frac{\partial\bar{C}}{\partial t}+\bar{V}\cdot\bar{\nabla}\bar{C}\right)=D_B\bar{\nabla}^2\bar{C}+\left(\frac{D_T}{\bar{T}_\infty}\right)\bar{\nabla}^2\bar{C}, \tag{4}$$

$$\bar{\nabla}\cdot\bar{j}=0, \tag{5}$$

$$\bar{j}=\bar{n}\bar{v}+\bar{n}v^{\simeq}-D_m\bar{\nabla}\bar{n}.$$

Here are ρ_f the density of the base fluid, k the thermal conductivity, μ the viscosity of the fluid, β the volume expansion coefficient and ρ_p the density of the nanoparticles. The gravitational acceleration is denoted by g. The coefficients that appear in Eqs. (3), (4) and (5) are the Brownian diffusion coefficient, the thermophoretic diffusion coefficient and the diffusivity of microorganisms respectively.

The boundary conditions are taken to be

$$u=U_w(x)=ax,\quad v=0,\quad \bar{T}=\bar{T}_w,\quad \bar{C}=\bar{C}_w,\quad \bar{n}=\bar{n}_w\quad at\ y=0,$$
$$u\to 0,\quad v\to 0,\quad T\to T_\infty,\quad \bar{C}\to\bar{C}_\infty,\quad \bar{n}\to\bar{n}_\infty\quad as\ y\to\infty. \tag{6}$$

In keeping with the Oberbeck–Boussinesq approximation and an assumption that the nanoparticle concentration is dilute, and with a suitable choice for the reference pressure, we can linearize the momentum equation and write Eq. (2) as

$$\rho_f\left(\frac{\partial\bar{V}}{\partial t}+\bar{V}.\,\bar{\nabla}\bar{V}\right)=-\bar{\nabla}p+\mu\bar{\nabla}^2\bar{V}+J\times B+[\left(\rho_p-\rho_f\right)\left(\bar{C}-\bar{C}_\infty\right)-\left(1-\bar{C}_\infty\right)\rho_f\beta(\bar{T}-\bar{T}_\infty)+\left(\bar{n}-\bar{n}_\infty\right)\Delta p\gamma]g, \tag{7}$$

We now make the standard boundary layer approximation, based on a scale analysis, and write the governing equations

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \tag{8}$$

$$0=\nu\left(\frac{\partial^2u}{\partial y^2}\right)-\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)-\frac{\sigma B_0^2}{\rho_f}-\left(\rho_p-\rho_f\right)\left(\bar{C}-\bar{C}_\infty\right)g+g\left(1-\bar{C}_\infty\right)\rho_f\beta(\bar{T}-\bar{T}_\infty)-\left(\bar{n}-\bar{n}_\infty\right)\Delta\rho\gamma g, \tag{9}$$

$$u\frac{\partial\bar{T}}{\partial x}+v\frac{\partial\bar{T}}{\partial y}=\alpha\frac{\partial^2\bar{T}}{\partial y^2}+\tau\left\{D_B\left(\frac{\partial\bar{C}}{\partial y}\frac{\partial\bar{T}}{\partial y}\right)+\left(\frac{D_T}{T_\infty}\right)\left(\frac{\partial\bar{T}}{\partial y}\right)^2\right\}, \tag{10}$$

$$u\frac{\partial\bar{C}}{\partial x}+v\frac{\partial\bar{C}}{\partial y}=D_B\frac{\partial^2\bar{C}}{\partial y^2}+\left(\frac{D_T}{T_\infty}\right)\frac{\partial^2\bar{T}}{\partial y^2}, \tag{11}$$

$$u\frac{\partial\bar{n}}{\partial x}+v\frac{\partial\bar{n}}{\partial y}+\frac{bW_c}{\left(\bar{C}_w-\bar{C}_\infty\right)}\frac{\partial}{\partial y}\left(\bar{n}\frac{\partial\bar{C}}{\partial y}\right)=D_m\frac{\partial^2\bar{n}}{\partial y^2}. \tag{12}$$

Introducing the following similarity transformations

$$\eta=\sqrt{\frac{a}{\nu}}y,\quad u=axf'(\eta),\quad v=-\sqrt{a\nu}f(\eta),\quad \theta(\eta)=\frac{\bar{T}-\bar{T}_\infty}{\bar{T}_w-\bar{T}_\infty},$$
$$\chi(\eta)=\frac{\bar{n}-\bar{n}_\infty}{\bar{n}_w-\bar{n}_\infty},\quad \phi(\eta)=\frac{\bar{C}-\bar{C}_\infty}{\bar{C}_w-\bar{C}_\infty}. \tag{13}$$

Making use of Eqs. (13) in Eqs. (8)–(12), we have

$$f'''-(f')^2+ff''-M^2f'+G_r(\theta-N_r\gamma-R_b\chi)=0, \tag{14}$$

$$\theta''+\Pr[f\theta'+N_b(\theta'\phi')+N_t(\theta')^2]=0, \tag{15}$$

$$\phi''+\Pr L_e(f\phi')+\frac{N_t}{N_b}\theta''=0, \tag{16}$$

$$\chi''+\Pr L_b(f\chi')-P_e\left(\phi'\chi'+\phi''\left(\sigma_1+\chi\right)\right)=0, \tag{17}$$

$$f(0)=0,\quad f'(0)=1,\quad f'(\infty)=0, \tag{18}$$

$$\theta(0)=1,\quad \theta(\infty)=0, \tag{19}$$

$$\phi(0)=1,\quad \phi(\infty)=0. \tag{20}$$

$$\chi(0)=1,\quad \chi(\infty)=0. \tag{21}$$

where $\Pr=\nu/\alpha$ is the Prandtl number. Expressions for the skin friction coefficient C_f, local Nusselt number Nu_x, local Sherwood number Sh_x and the local density number of the motile microorganisms Nn_x which are defined as

$$C_f=\frac{\tau_w}{\rho U_w^2},\quad \mathrm{Nu}_x=\frac{x\bar{q}_w}{\alpha(\bar{T}_w-\bar{T}_\infty)},\quad \mathrm{Sh}_x=\frac{x\bar{q}_m}{D_B(\bar{C}_w-\bar{C}_\infty)},$$
$$Nn_x=\frac{x\bar{q}_n}{D_B(\bar{n}_w-\bar{n}_\infty)}, \tag{22}$$