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A differential algebraic approach for the modeling of polycrystalline ferromagnetic hysteresis with minor loops and frequency dependence

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ABSTRACT

In the current paper, a nonlinear differential algebraic approach is proposed for the modeling of hysteretic dynamics of polycrystalline ferromagnetic materials. The model is constructed by employing a phenomenological theory to the magnetization orientation switching. For the modeling of hysteresis in polycrystalline ferromagnetic materials, the single crystal model is applied to each magnetic domain along its own principal axis. The overall dynamics of the polycrystalline materials is obtained by taking a weighted combination of the dynamics of all magnetic domains. The weight function for the combination is taken as the distribution function of the principal axes. Numerical simulations are performed and comparisons with its experimental counterparts are presented. The hysteretic dynamics caused by orientation switching processes is accurately captured by the proposed model. Minor hysteresis loops associated with partial-amplitude loadings are also captured. Rate dependence of the hysteresis loops are inherently incorporated into the model due to its differential nature.

1. Introduction

Hysteresis is a common phenomenon existing in most magnetic materials and other smart materials, if not all. Static hysteresis loop in the H–M curve (H is the magnetic field, M is the magnetization) was found centuries ago, and its correspondence with the magnetization orientation switching was well accepted [1–3]. At the same time, in other materials such as electromagnetic, ferromagnetic, and magnetoelastic materials, hysteretic behaviors of the magnetic fields have a dramatic influence on the dynamics of other physical fields involved in the systems, due to the complicated coupling effects among them.

Modeling of hysteresis loops in the H–M curves is a classic problem and has attracted a lot attention. Among the models employed for hysteresis investigations, the Preisach model [1,2,4] and the Jiles-Atherton model [5] are the two main models. The Preisach model was first proposed by Preisach based on some assumption of the physical mechanism of magnetization phenomenon in 1935. The basic idea of the model is that magnetic material is assumed to consist of a group of magnetic dipoles which have rectangle hysteric behaviors. Macroscopic hysteric behavior of the material is seen as the sum of hysteric behaviors of these magnetic dipoles and the system output is the weighted sum of all basic hysteresis relay operators in material. In the 1970s, the Russian mathematician Krasnoselskii separated this model from its physical meaning and represented it in a pure mathematical form which is similar to a spectral resolution of operators [6]. The new generalized Preisach model, which is a mathematical model and belongs to phenomenological theory system because of its independence of physical mechanism of hysteresis, can now be used for the mathematical description of hysteresis of any physical nature. The advantage of the mathematical model is that it ensures the closure of the minor hysteresis loop and can be applied to the condition where physical mechanism hasnot been well understood because of its generalized algorithm. It also has high predictive ability of nonlinear hysteresis and good versatility. But the model can represent the relationship only between the input and output and provides no insights into the physical mechanism of hysteresis dynamics of the system. In addition, the complex formulas and the identification of a large number of non-physical parameters are needed which make the model inflexible and time-consuming. Currently there are still many efforts devoted to modifying and improving the Preisach model for the better modeling of the hysteresis dynamics behavior [7].

In 1986, the J-A model was proposed by Jiles and Atherton based on the ferromagnetic theory which believe that the irreversible domain wall motion results in the hysteresis. The effective magnetic field in material is the sum of the excitation field, the interaction field between domains and the field induced by stress, and the magnetization consists of reversible and irreversible

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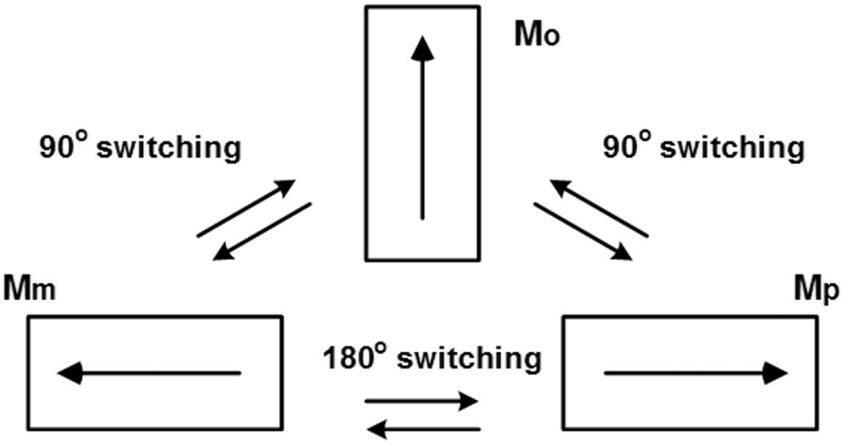

Fig. 1. Sketch of magnetization orientation switching in a one-dimensional analog.

components. The J-A model which is represented by a differential equation can't ensure the closure of the minor hysteresis loop although it has been proved to have good precision on modeling the hysteretic behavior in magnetostrictive material when the change of the magnetic field keeps at a constant rate. In 1992, the original model was extended by Jiles to make the minor hysteresis loop close [8], but this reduces the utility of the model for dynamic control. Furthermore, the J-A model can capture the hysteric behavior only in ferromagnetic materials.

Motivated by engineering applications, the model for hysteresis is desired to incorporate the dynamical behavior of the orientation switching processes, which is responsible for the dependence of the hysteresis loops on the loading rates. At the same time, it should also be capable of capturing minor hysteresis loops caused by partial-amplitude loading cycles [3,9]. Meanwhile, it is also beneficial to construct a differential model for the hysteretic dynamics since classic control and optimization theory are well established on the basis of differential equations. Recently, a unified framework was proposed for the modeling of dynamics involving hysteresis in ferroric materials [9]. The approach was based on the thermodynamic theory and statistic mechanics and the model was formulated as ordinary differential equations. But the application of the model in control systems is not a trivial task at all because it is computationally very expensive [3,9].

In this paper, a macroscopic differential model for the nonlinear dynamics of ferromagnetic materials is proposed on the basis of modeling the magnetization orientation switching, in a one dimensional description. The essence of the model is to associate the hysteretic dynamics to the orientation switching dynamics in the materials. Hysteretic dynamics of single crystal materials is first modeled by using the Landau theory of phase transition [10]. For the modeling of the hysteretic dynamics of polycrystalline materials, the single crystal model is extended by using an assumption that the principal axes of different magnetization domains have a different direction, and has a certain distribution. The hysteretic dynamics in each magnetic domain can be modeled as a single crystal case in its own principal axis direction. The overall dynamics is then modeled by taking a weighted combination of the dynamics of all involved domains [11]. It is shown by comparison with experimental results that the hysteresis dynamics related to magnetization orientation switching can be accurately captured by the proposed differential model. The rate dependence property of the hysteresis loops is inherently incorporated due to the facts that the model is given by ordinary differential equations. Minor hysteresis loops are also successfully captured.

2. Magnetization orientation switching

It is well understood nowadays that the hysteretic behavior of ferromagnetic materials is a consequence of orientation switching of magnetic dipoles (moments) upon employing magnetic fields. If the discussion is confined within a one-dimensional description for the sake of clarification and the material temperature is assumed to be below the Curie temperature, the magnetic dipoles will have two orientations (parallel to its principal axis), and all switchings induced by magnetic fields are 180°. In other cases, dipoles can also be switched 90°, which makes the new orientation perpendicular to the original orientations. To account for the effects of 90° switching in the one dimensional analog, an extra orientation which is perpendicular to the principal axis is introduced here. The perpendicular orientation has no contribute of magnetization to the principal axis direction, but it will change the orientation switching dynamics in many cases [11,12].

For the convenience of the following discussion, the rightward orientation of the magnetic dipoles is denoted as M_p while the leftward one as M_m. The one perpendicular to the principal axis is denoted as M_o. Upon employing external magnetic fields, orientation switching can be induced between the two opposite orientations directly ($M_p \leftrightarrow M_m$). At the same time, 90° orientation switching could also be induced ($M_o \leftrightarrow M_p$ or $M_o \leftrightarrow M_m$). A sketch of the orientation switching in a one-dimensional analog is presented in Fig. 1. Hysteresis is then induced by the orientation switching between these three orientations.

According to the Landau theory, the essential element in the modeling of phase transition dynamics is a free energy function characterizing different phases involved [10]. Here, there are three orientations involved, the free energy function can be constructed as a polynomial retaining the 6th order term of the order parameter [10], such that it is capable of providing three local minima, while being an even function to account for the symmetry properties in physics. In order to construct a macroscale differential model, the potential energy function here is constructed as the same as the Landau free energy function, as follows:

$$F(M) = \frac{a_2}{2}M^2 + \frac{a_4}{4}M^4 + \frac{a_6}{6}M^6 \tag{1}$$

where a_2, a_4, and a_6 are material constants, M is the magnetization which is chosen as the only order parameter. For a specified material, the material constants could be determined by experimental tests. It should be realized that the magnetostatic interaction and the exchange interaction have already incorporated into the Landau free energy function. In Ref. [13], R.C. Smith etc. have deduced the Helmholtz free Energy, which has a similar profile with