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Re-direction of dc magnetic flux in magnetically isotropic multilayered structures

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ABSTRACT

Analytical design of a periodic composite structure allowing re-direction (bending) of dc magnetic flux with respect to applied external field is presented using methods of transformation optics. The composite structure is made of micrometer scale alternating layers of two different homogeneous and magnetically isotropic materials. Dependence of the magnetic flux bending angle on geometrical orientation of the layers as well as on the magnetic permeability ratio is examined. Such structures can find use in various devices based on the control and manipulations of the magnetic flux.

1. Introduction

In recent years, methods of transformation optics [1–4] have been widely used to design many devices of various type controlling not only the propagation of electromagnetic waves but also indicating manipulation of heat and electric currents [5–12], acoustic [13] and material [14] waves. One of the most exotic and up-to-date applications of transformation electrodynamics consists in the realization of metamaterial devices based on cloaking and concentration effects for a static magnetic field [15–20]. Obviously, similar effects can open new perspectives for manipulation of the magnetic flux and applications in different fields such as magnetic sensing, mass spectrometers, charged particle accelerators, etc.

The aim of this letter is an analytical design of a multilayered composite structure allowing re-direction (bending) of dc magnetic flux with respect to the applied external field. Individual layers of the composite under consideration are described by isotropic magnetic susceptibilities so as the magnetic flux density is always parallel to the applied magnetic field intensity.

It is evident that bending of the magnetic flux is only possible if the whole structure possess anisotropic magnetic permeability. We will study a periodic multilayer structure consisting of alternating layers of two different materials with isotropic permeability μ_1 and μ_2 (let $\mu_1 > \mu_2$) of thickness l_1 and l_2 in micrometer scale, so as $l_{1,2}$ are much larger than the lattice constants of the constituent components. The most common techniques for the tailoring artificial periodicity with the precise control of layer thickness are molecular-beam epitaxy [21] and metal-organic chemical vapor deposition [22].

2. Model and Theory

The multilayered periodic structure can be considered as an effective anisotropic homogeneous medium (a metamaterial) with magnetic flux density vector

$$\mathbf{B} = \mu_0 \hat{\mu}_{ef} \mathbf{H}, \tag{1}$$

where μ_0 is magnetic permeability of the free space and $\hat{\mu}_{ef}$ is the relative effective permeability tensor of the composite structure. Neglecting interfacial effects and using definitions of harmonic and arithmetic means, it is not difficult to show that in the coordinate system with the x-axis perpendicular to the plane of the layers (Fig. 1) tensor $\hat{\mu}_{ef}$ has the form

$$\hat{\mu}_{ef} = \begin{pmatrix} \mu_{xx} & 0 & 0 \\ 0 & \mu_{yy} & 0 \\ 0 & 0 & \mu_{zz} \end{pmatrix}, \tag{2}$$

where

$$\mu_{xx} = \frac{\mu_1 \mu_2 d}{\mu_1 l_2 + \mu_2 l_1}, \quad \mu_{yy} = \mu_{zz} = (\mu_1 l_1 + \mu_2 l_2)/d, \tag{3}$$

and $d = l_1 + l_2$ is the period of the structure.

Note that the criterion for an effective medium in the case of electromagnetic interaction is the period/wavelength ratio of 1:10 or less. As a static field is the limiting case when the wave frequency

$\nu \to 0$ in Maxwell equations, i.e., the wavelength $\lambda \to \infty$, this criterion is always fulfilled including the case when l_1 and l_2 are in micrometer scale. Note also that transversal (with respect to the layers) component μ_{xx} is always less than the longitudinal components μ_{yy} and μ_{zz}.

An additional anisotropy in the permeability can be introduced rotating the layers in the structure around the z-axis on an angle φ (see Fig. 2). Such a rotation can be realized using coordinate transformations

$$x' = x \cos\varphi + y \sin\varphi, \quad y' = -x \sin\varphi + y \cos\varphi, \quad z' = z, \tag{4}$$

where $x'y'z'$ is the reference coordinate system fixed to the sample

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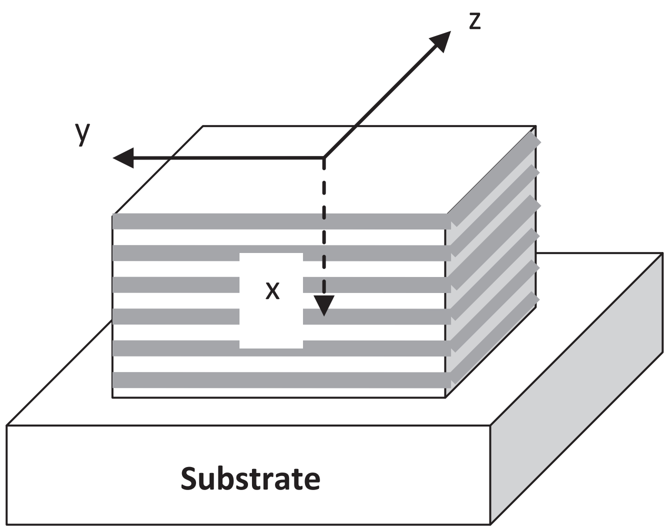

Fig. 1. Geometry of anisotropic periodic structure with layers perpendicular to the x-axis.

of rectangular parallelepiped form while xyz system of the co-ordinate axes is connected with the layers which are arranged in the direction perpendicular to the x-axis. The angle φ is assumed to be positive if the rotation is in counter-clockwise direction.

The static magnetic field in a source free space is described by Maxwell equations [23]

$$\textbf{curl}\,\mathbf{H} = 0, \quad \mathrm{div}\left(\hat{\mu}_{ef}\mathbf{H}\right) = 0 \tag{5}$$

To keep Eq. (5) form-invariant under transformations (4), the effective permeability tensor should be transformed as [1]

$$\hat{\mu}' = \frac{\hat{J}\,\hat{\mu}_{ef}\hat{J}^T}{\det|\hat{J}\,|}, \tag{6}$$

where $\hat{J}$ is the Jacobian transformation matrix

$$\hat{J} = \begin{pmatrix} \cos\varphi & \sin\varphi & 0 \\ -\sin\varphi & \cos\varphi & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{7}$$

and $\hat{J}^T$ is the transpose of $\hat{J}$. Then for the effective permeability tensor in the coordinate system $x'y'z'$ we obtain

$$\hat{\mu}' = \begin{pmatrix} \mu_{xx}\cos^2\varphi + \mu_{yy}\sin^2\varphi & (\mu_{yy} - \mu_{xx})\sin\varphi\cos\varphi & 0 \\ (\mu_{yy} - \mu_{xx})\sin\varphi\cos\varphi & \mu_{xx}\sin^2\varphi + \mu_{yy}\cos^2\varphi & 0 \\ 0 & 0 & \mu_{zz} \end{pmatrix}. \tag{8}$$

Let the external magnetic field $\mathbf{B_0}{=}\mu_0\mathbf{H_0}$ is applied along the x' axis. Then, using Eqs. (3) and (8) one can conclude that the magnetic flux density vector $\mathbf{B}^{/}\mu_0\hat{\mu}'\mathbf{H_0}$. in the composite is bending with respect to $\mathbf{B_0}$ on the angle θ, where

$$\tan\theta = \frac{\mu_{y,x'}}{\mu_{x,x'}} = \frac{\beta\tan\varphi}{1 + (1+\beta)tan^2\varphi}, \tag{9}$$

and

$$\beta \equiv l_1 l_2(\mu_1 - \mu_2)^2/\mu_1\mu_2 d^2. \tag{10}$$

Note that θ is an odd function of φ: $\theta(-\varphi) = -\theta(\varphi)$. It is easily to see that for all values of φ in the range $0 < \varphi < \pi/2$, the bending angle θ is less than φ, if $\beta \le 1$. In the case when $\beta > 1$, $\theta > \varphi$ if $\varphi < \varphi_0$ and $\theta \le \varphi$ if $\varphi \ge \varphi_0$, where

$$\varphi_0 = \frac{1}{2}cos^{-1}\left(\frac{1}{\beta}\right) = tan^{-1}\left(\frac{\beta - 1}{\beta + 1}\right)^{1/2}. \tag{11}$$

Obviously, the angle φ_0 at which $\theta = \varphi = \varphi_0$ is always less than $\pi/4$.

3. Discussion and results

Before to examine the dependence of the bending angle on the orientation angle φ in detail, it is convenient to rewrite the expression for the material parameter β in the form

$$\beta \equiv \frac{\alpha(1-\gamma)^2}{\gamma(1+\alpha)^2}, \tag{12}$$

where

$$\alpha \equiv l_2/l_1,\ \gamma \equiv \mu_2/\mu_1 \tag{13}$$

and γ changes in the region $0 \le \gamma < 1$. For simplicity, we will further restrict ourselves to the consideration of the case when different layers of the structure have the same thickness: $\alpha = 1$ and

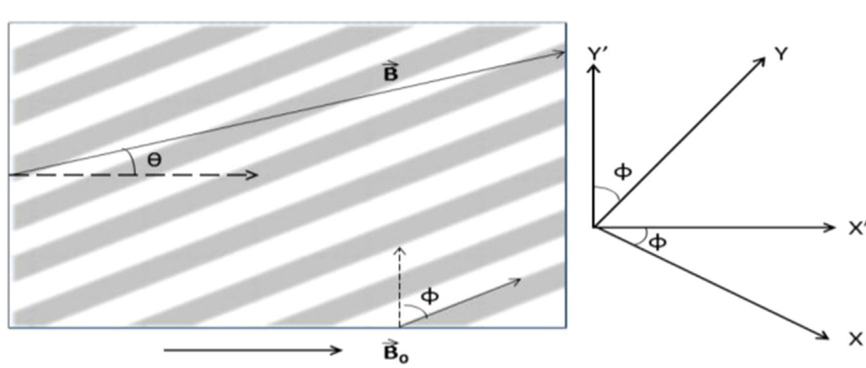

Fig. 2. Cross-section of the multilayer composite in the $x'y'$-plane with layers rotated on angle φ around the z-axis; θ is the bending angle of the magnetic flux density.