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Effects of the fractional order and magnetic field on the blood flow in cylindrical domains

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ABSTRACT

In this paper, based on the magnetohydrodynamics approach, the blood flow along with magnetic particles through a circular cylinder is studied. The fluid is acted by an oscillating pressure gradient and an external magnetic field. The study is based on a mathematical model with Caputo fractional derivatives. The model of ordinary fluid, corresponding to time-derivatives of integer order, is obtained as a particular case. Closed forms of the fluid velocity and magnetic particles velocity are obtained by means of the Laplace and finite Hankel transforms. Effects of the order of Caputo's time-fractional derivatives and of the external magnetic field on flow parameters of both blood and magnetic particles are studied. Numerical simulations and graphical illustrations are used in order to study the influence of the fractional parameter α, Reynolds number and Hartmann number on the fluid and particles velocity. The results highlights that, models with fractional derivatives bring significant differences compared to the ordinary model. This fact can be an important advantage for some practical problems. It also results that the blood velocity, as well as that of magnetic particles, is reduced under influence of the exterior magnetic field.

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1. Introduction

Biomagnetic fluids play an important role in various areas of bioengineering and medical sciences. Most of the interests regarding the biomagnetic fluids include the development of magnetic devices for cell separation, transport of drugs using magnetic particles, treatment of cancer tumors causing magnetic hyperthermia, and reduction of bleeding during surgeries [1–3]. Biomagnetic fluid dynamics is a new branch in fluid mechanics which investigates behavior of biological fluids under the influence of magnetic field. The blood is considered to be the most characteristic biomagnetic fluid. It is a magnetic fluid because, the erythrocytes have hemoglobin molecules which are oxides of iron and are present with exclusively high concentration in the mature red blood cells. The blood magnetic property is affected by the state of oxygenation [4].

Haik et al. [5] have developed a mathematical formulation of biofluid dynamics which is analogous to the one of the ferrohydrodynamics. In this theory the induced electric current is neglected and, the flow is affected by the magnetization of the fluid under the influence of magnetic field. Several researchers studied the blood flow using magnetohydrodynamics, which studies electrical conducting fluids, by including the Lorenz force due to induced currents and ignores the effect of magnetization [6].

Fractional calculus, involving derivatives and integrals of non-integer order, is a generalization of the classical calculus. In the last years, it is widely used for modeling and control of processes in many areas of sciences and engineering [7,8]. In many areas, such as mechanics, chemistry, bioengineering, the fractional calculus provides more accurate models of the physicals systems than the ordinary calculus. The memory and hereditary properties of the materials can also be described by means of the fractional derivatives. Mathematical models with fractional derivatives are also used in the fractal theory, electrical circuits and in the electromagnetic theory [9,10]. Other interesting topics can be found in papers [11–14].

By using the magnetohydrodynamic approach, Sharma et al. [4] have studied the fluid flow parameters of blood along with magnetic particles in a cylindrical tube, under influence of a constant magnetic field and of an oscillating pressure gradient in the axial

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direction. Solution of initial-boundary value problem was obtained by means of the finite difference approximation. For comparison, the effect of external magnetic field on flow parameters of both blood and magnetic particles was experimentally studied using the artificial blood (75 percentage water, 25 percentage glycerol) along with iron oxide magnetic particles.

In this paper we study a blood model with fractional derivatives, starting from the classical model studied by Sharma et al. [4]. Specifically, we study the blood flow along with magnetic particles through a cylindrical tube under the influence of a magnetic field perpendicular to the direction of flow and an oscillating pressure gradient in the axial direction. The model with fractional derivatives is obtained from the classical model by replacing the time derivative of order one with the Caputo fractional derivative [15,16].

Closed forms of both velocities, of blood and magnetic particles, are obtained by means of the finite Hankel and Laplace transforms. These solutions are easily particularized for the classical fluid (the fractional parameter $\alpha = 1$) and for the clean fluid. The influence of the fractional parameter and magnetic field on the fluid velocity and the particles velocity was studied by means of numerical simulations using the Mathcad software. The positive roots of the Bessel function $J_o(x)$, which are necessary in the numerical study, were also generated with the same software. Based on numerical calculations the diagrams of fluid and particles velocities for various values of fractional and material parameters have been plotted. Results show that the time fractional derivative has a significant influence on the flow parameters, namely the fluid modeled by fractional derivatives flows slower or faster than the ordinary fluid ($\alpha = 1$). Also, with a fractional model it is possible to obtain, at a certain time, a given velocity in a position r. This may be advantageous for some practical problems. Our results highlights that, the fluid velocity and the velocity of magnetic particles are reduced by a stronger magnetic field. The magnetic particles are moving in the same trend like blood but with smaller velocity.

2. Formulation of the problem

We consider the blood flow along with magnetic particles in a circular cylindrical tube of radius R_o (Fig. 1). The magnetic particles are supposed to be uniformly distributed throughout the blood.

The blood and magnetic particles are flowing in the axial direction denoted by z while the applied magnetic field acts perpendicular to the flow direction. In this paper we consider that the magnetic Reynolds number is very small such that the induced magnetic field effect can be neglected [17]. At the initial moment $t=0$, the tube , blood and particles are at rest. If a magnetic material flows in a magnetic field, it experiences an electromotive force which results to flow an electric current. By applying a magnetic field on an electrically conducting fluid like blood, an electromagnetic force is generated due to the interaction of current with the magnetic field. The electromotive force depends on the speed of motion as well as magnetic flux intensity [4,18].

The problem into consideration involves Navier–Stokes equations describing the fluid flow, Maxwell's relations for magnetic field and Newton's second law for the particles motion. Maxwell equations are

$$\nabla . \vec{B} = 0, \nabla \times \vec{B} = \mu_o \vec{J}, \nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}, \tag{1}$$

where $\vec{B}$ is the magnetic flux intensity, μ_o is the magnetic permeability, $\vec{E}$ is the electric field intensity and $\vec{J}$ is the current density given by

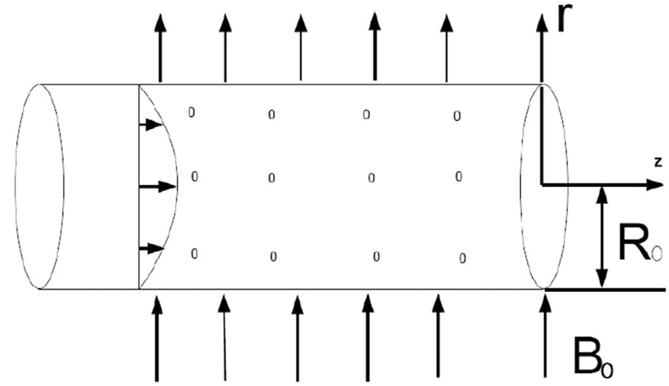

Fig. 1. Flow geometry.

$$\vec{J} = \sigma(\vec{E} + \vec{V} \times \vec{B}), \tag{2}$$

with σ being the electrical conductivity and $\vec{V}$ being the velocity field.

The electromagnetic force $\vec{F}_{em}$ is defined as

$$\vec{F}_{em} = \vec{J} \times \vec{B} = \sigma(\vec{E} + \vec{V} \times \vec{B}) \times \vec{B} = -\sigma B_o^2 u(r, t)\vec{k}, \tag{3}$$

where $\vec{k}$ is the unite vector of the z-direction and $\vec{V} = u(r, t)\vec{k}$ is the axial velocity of the blood. The force $\vec{F}_{em}$ will be included in the momentum equations.

The unsteady flow of blood in an axisymmetric cylindrical tube of radius R_o is considered under the influence of uniform transverse magnetic field and a pressure gradient of the form [19]:

$$-\frac{\partial p}{\partial z} = A_o + A_1 \cos(\omega t), \quad A_o > 0. \tag{4}$$

where the constant A_o and A_1 are the amplitudes of the pressure gradient, respectively of the pulsatile component giving rise to systolic or diastolic pressure.

The governing momentum equation for fluid flow in cylindrical coordinates (r, θ, z) is given by [4,13]

$$\frac{\partial u}{\partial t} = -\frac{1}{\rho}\frac{\partial p}{\partial z} + \nu\left(\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r}\right) + \frac{KN}{\rho}(v - u) - \frac{\sigma B_o^2}{\rho}u, \tag{5}$$

where ρ is the fluid density, ν is the kinematic viscosity , p is the pressure, N is the number of magnetic particles per unite volume, K is the Stokes constant, u is the fluid velocity and v is the velocity of the particle. The term $\frac{KN}{\rho}(v - u)$ is the force due to the relative motion between fluid and magnetic particles. It is assumed that the Reynolds number of the relative velocity is small. In such a case the force between magnetic particles and blood is proportional to the relative velocity.

The motion of magnetic particles is governed by Newton's second law

$$m\frac{\partial v}{\partial t} = K(u - v), \tag{6}$$

where m is the average mass of the magnetic particles.

In order to consider the time-fractional model, we firstly multiply Eqs. (5) and (6) by $\lambda = \sqrt{\frac{R_o\rho}{A_o}}$, which has the dimension of the time t. The governing equations of the time-fractional model are

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