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On magnetohydrodynamic flow of second grade nanofluid over a nonlinear stretching sheet

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ABSTRACT

This research article addresses the magnetohydrodynamic (MHD) flow of second grade nanofluid over a nonlinear stretching sheet. Heat and mass transfer aspects are investigated through the thermophoresis and Brownian motion effects. Second grade fluid is assumed electrically conducting through a non-uniform applied magnetic field. Mathematical formulation is developed subject to small magnetic Reynolds number and boundary layer assumptions. Newly constructed condition having zero mass flux of nanoparticles at the boundary is incorporated. Transformations have been invoked for the reduction of partial differential systems into the set of nonlinear ordinary differential systems. The governing nonlinear systems have been solved for local behavior. Graphical results of different influential parameters are studied and discussed in detail. Computations for skin friction coefficient and local Nusselt number have been carried out. It is observed that the effects of thermophoresis parameter on the temperature and nanoparticles concentration distributions are qualitatively similar. The temperature and nanoparticles concentration distributions are enhanced for the larger magnetic parameter.

1. Introduction

The homogeneous mixture of ultrafine nanoparticles and base fluid is known as nanofluid. The nanoparticles are typically made of metals (Al, Cu, Ag) or nonmetals (graphite, carbon nanotubes) and the base fluid is commonly a conductive fluid such as water, oil or ethylene glycol. The suspended nanoparticles are capable to enhance the thermal conductivity and heat transfer performance because the thermal conductivity of solid metals is higher than the base fluids. Nanofluids have several engineering and technological applications such as cooling of electronic devices, vehicle cooling, heat exchanger, nuclear reactor, vehicle thermal management and many others. Especially the magneto nanofluids are helpful in wound treatments, removal of blockage in the arteries, cancer therapy, hyperthermia, magnetic resonance imaging and many others. Choi [1] introduced the term nanofluid and illustrated that the suspension of nanoparticles increases the thermal properties of base liquids. Then Buongiorno [2] developed a mathematical model of nanofluid which exhibits the characteristics of Brownian motion and thermophoresis. Khan and Pop [3] explored the boundary-layer flow of nanofluid over a linear stretching surface. Boundary-layer flow of nanofluid over a linear stretching surface subject to the convective boundary condition is investigated by Makinde and Aziz [4]. Mustafa et al. [5] discussed the stagnation point flow of nanofluid induced by a linear stretching surface. Afterwards various attempts have been made in this direction. Few of these can be quoted through the investigations [6–20] and several refs. therein.

The boundary-layer flow over a stretching surface is important in various industrial and technological processes like paper production, hot rolling, wire drawing, glass fiber, extrusion of plastic sheets, drawing of plastic films and many others. The classical problem of two-dimensional (2D) flow induced due to a non-linear stretching sheet was addressed by Vajravelu [21]. In this work, the velocity of the sheet was assumed to obey the power law distribution, i.e. $u_w = cx^n$. Then Cortell [22] extended this problem by considering viscous dissipation and thermal radiation. Hayat et al. [23] examined the magnetohydrodynamic (MHD) flow over a nonlinear stretching surface by employing the modified Adomian decomposition and Pade approximation techniques. Rana and Bhargava [24] examined the flow and heat transfer characteristics of nanofluid due to a nonlinear stretching surface. Mukhopadhyay [25] analyzed the boundary layer flow over a permeable nonlinear stretching surface with partial slip condition. MHD flow and heat transfer of nanofluid due to a nonlinear stretching sheet is numerically addressed by Mabood et al. [26]. Mustafa et al. [27]

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discussed the axisymmetric flow of nanofluid over a nonlinear stretching sheet. They computed both analytical and numerical solutions of the resulting problems. Rashidi et al. [28] examined the Darcy–Forchheimer flow and heat transfer around an obstacle with transverse magnetic field effects.

The study of non-Newtonian fluids have great importance due to its various industrial and engineering applications. Particularly such fluids are encountered in the material processing, chemical and nuclear industries, oil reservoir engineering and foodstuffs, etc. Examples of non-Newtonian fluids are paints, shampoos, ketchup, applesauce, certain oils, polymer solutions and many others. All the non-Newtonian fluids through their distinct characteristics cannot be distinguished by using a single relationship. Hence various models have been proposed to describe the properties of non-Newtonian fluids. Usually non-Newtonian fluids are classified into three categories, namely (i) differential type (ii) rate type and (iii) integral type. The second grade fluid model [29–33] is the simplest subclass of differential type fluids. This model describes the effects of normal stress.

The purpose of the present communication is to develop a mathematical model for two-dimensional (2D) flow of second grade nanofluid. Flow here is induced by a nonlinear stretching surface. Fluid is conducted in the presence of non-uniform magnetic field. Thermophoresis and Brownian motion effects are considered. Newly proposed condition with the zero nanoparticles mass flux at the surface is taken into account. Mathematical formulation is presented subject to small magnetic Reynolds number and boundary layer assumptions. Solutions of non-dimensional velocity, temperature and nanoparticles concentration distributions are developed via the homotopy analysis method (HAM) [34–45]. Graphs are sketched to analyze the impacts of various influential parameters on the velocity, temperature and nanoparticles concentration distributions. Skin friction coefficient and local Nusselt number are tabulated and discussed.

2. Formulation

Consider the steady two-dimensional (2D) flow of an incompressible second grade nanofluid over a nonlinear stretching sheet. Thermophoresis and Brownian motion effects are incorporated. Second grade fluid is electrically conducted due to a non-uniform magnetic field applied in the $y-$ direction. Here the Hall current and electric field effects are not considered subject to small magnetic Reynolds number. We adopt the Cartesian coordinate system in such a way that $x-$ axis is assumed along the stretched sheet and $y-$ axis is perpendicular to the sheet. The sheet at $y = 0$ is stretched along the $x-$ direction with velocity $u_w(x) = ax^n$ where $a,\ n > 0$ are the constants. The boundary layer expressions governing the flow of second grade nanofluid are

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2} + k_0\left(u\frac{\partial^3 u}{\partial x\partial y^2} + \frac{\partial u}{\partial x}\frac{\partial^2 u}{\partial y^2} + v\frac{\partial^3 u}{\partial y^3} - \frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial x\partial y}\right) - \frac{\sigma B^2(x)}{\rho_f}u, \tag{2}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2} + \frac{(\rho c)_p}{(\rho c)_f}\left(D_B\left(\frac{\partial T}{\partial y}\frac{\partial C}{\partial y}\right) + \frac{D_T}{T_\infty}\left(\frac{\partial T}{\partial y}\right)^2\right), \tag{3}$$

$$u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y} = D_B\left(\frac{\partial^2 C}{\partial y^2}\right) + \frac{D_T}{T_\infty}\left(\frac{\partial^2 T}{\partial y^2}\right). \tag{4}$$

The subjected boundary conditions for the considered flow problem are

$$u = u_w(x) = ax^n,\quad v = 0,\quad T = T_w,\ D_B\frac{\partial C}{\partial y} + \frac{D_T}{T_\infty}\frac{\partial T}{\partial y} = 0 \quad \text{at} \quad y = 0, \tag{5}$$

$$u \to 0,\quad T \to T_\infty,\quad C \to C_\infty \quad \text{as} \quad y \to \infty. \tag{6}$$

Here u and v represent the flow velocities in the horizontal and vertical directions respectively, $\nu(=\mu/\rho_f)$ stands for kinematic viscosity, μ denotes the dynamic viscosity, ρ_f stands for density of base liquid, $k_0 = \alpha_1/\rho_f$ stands for elastic parameter, α_1 represents the normal stress moduli, σ stands for electrical conductivity, $B(x) = B_0 x^{\frac{n-1}{2}}$ denotes the non-uniform magnetic field, T stands for temperature, $\alpha = k/(\rho c)_f$ stands for thermal diffusivity of fluid, k denotes the thermal conductivity, $(\rho c)_f$ stands for heat capacity of liquid, $(\rho c)_p$ stands for effective heat capacity of nanoparticles, D_B stands for Brownian diffusivity, C stands for nanoparticles concentration, D_T stands for thermophoretic diffusion coefficient, T_∞ stands for ambient fluid temperature, C_∞ denotes the ambient fluid concentration and a represents the rate of stretching. We now use the following transformations

$$\left.\begin{aligned} &u = ax^n f'(\eta),\quad v = -\left(\tfrac{a\nu(n+1)}{2}\right)^{1/2} x^{\frac{n-1}{2}}\left(f + \tfrac{n-1}{n+1}\eta f'\right),\\ &\theta(\eta) = \tfrac{T-T_\infty}{T_w - T_\infty},\quad \phi(\eta) = \tfrac{C-C_\infty}{C_\infty},\quad \eta = \left(\tfrac{a(n+1)}{2\nu}\right)^{1/2} x^{\frac{n-1}{2}} y. \end{aligned}\right\} \tag{7}$$

Eq. (1) is now automatically satisfied and Eqs. (2)–(6) lead to the following system

$$f''' + ff'' - \frac{2n}{n+1}(f')^2 + K\begin{pmatrix} (3n-1)f'f''' \\ -\left(\frac{n+1}{2}\right)ff^{(\text{iv})} - \left(\frac{3n-1}{2}\right)(f'')^2 \end{pmatrix} - M^2 f' = 0, \tag{8}$$

$$\theta'' + \Pr\left(f\theta' + Nb\theta'\phi' + Nt\theta'^2\right) = 0, \tag{9}$$

$$\phi'' + Le\Pr f\phi' + \frac{Nt}{Nb}\theta'' = 0, \tag{10}$$

$$f = 0,\ f' = 1,\quad \theta = 1,\ Nb\varphi' + Nt\theta' = 0 \quad \text{at} \quad \eta = 0, \tag{11}$$

$$f' \to 0,\ \theta \to 0,\ \varphi \to 0 \quad \text{as} \quad \eta \to \infty. \tag{12}$$

In the above expressions K denotes the local second grade parameter, M represents the magnetic parameter, Pr stands for Prandtl number, Nb depicts the Brownian motion parameter, Nt stands for thermophoresis parameter and Le stands for Lewis number. These parameters are defined by

$$\left.\begin{aligned} &K = \tfrac{\alpha_1 a x^{n-1}}{\mu},\ M^2 = \tfrac{2\sigma B_0^2}{a\rho_f(n+1)},\ \Pr = \tfrac{\nu}{\alpha},\\ &Nb = \tfrac{(\rho c)_p D_B C_\infty}{(\rho c)_f \nu},\ Nt = \tfrac{(\rho c)_p D_T (T_w - T_\infty)}{(\rho c)_f \nu T_\infty},\ Le = \tfrac{\alpha}{D_B}. \end{aligned}\right\} \tag{13}$$

The hydrodynamic flow situation is recovered for $M = 0$. Dimensionless forms of skin friction coefficient and the local Nusselt number can be written as follows:

$$\left.\begin{aligned} &Re_x^{1/2} C_f = \left(\tfrac{n+1}{2}\right)^{1/2}\left(1 + K\left(\tfrac{7n-1}{2}\right)\right) f''(0),\\ &Re_x^{-1/2} Nu_x = -\left(\tfrac{n+1}{2}\right)^{1/2}\theta'(0). \end{aligned}\right\} \tag{14}$$