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Characteristics of magnetic field and melting heat transfer in stagnation point flow of Tangent-hyperbolic liquid

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ABSTRACT

This paper examines the influence of melting heat transfer in the stagnation point flow of an incompressible magnetohydrodynamic (MHD) Tangent hyperbolic fluid. Stretched flow by a vertical surface is considered. Inclined nature of magnetic field is taken for an electrically conducting liquid. The resulting non-linear differential systems are computed for the convergent series solutions. Influences of various pertinent parameters like Weissenberg, magnetic, melting, ratio, angle of inclination, mixed convection, Eckert and Prandtl on the velocity and temperature are analyzed. Numerical data for various parameters on skin friction coefficient and local Nusselt number is also examined. It is found that the melting parameter reduces the temperature and thermal boundary layer while it shows opposite behavior for the velocity. Mixed convection has different role in the assisting and opposing flows.

1. Introduction

Recent developments of modern technologies have stimulated interest in fluid flows when forced and free convection act simultaneously. Mixed convection phenomenon occurs in which the effect of buoyancy forces on a forced flow is significant. The problem of mixed convection due to moving surfaces has wide applications in material processing systems and engineering devices such as welding, extrusion of plastics, hot rolling, paper drying, atmospheric boundary layer flows, heat exchangers, solar collectors, nuclear reactors and electronic equipments. Imtiaz et al. [1] investigated the mixed convection flow of nanofluid in the presence of Newtonian heating. Hayat et al. [2] analyzed the mixed convection flow of viscoelastic fluid towards a vertical stretching cylinder. Rashidi et al. [3] reported the problem of mixed convection flow of micropolar fluid towards a heated shrinking surface. Lok et al. [4] examined an axisymmetric mixed convection stagnation point flow towards a stretching or shrinking cylinder. Free convective heat and mass transfer in steady magnetohydrodynamic fluid flow over a stretching vertical surface in porous medium is examined by Rashidi et al. [5]. Turkyilmazoglu and Pop [6] considered the heat and mass transfer characteristics of some nanofluid flows past a vertical flat plate. They discussed the radiation effect for two distinct types of thermal boundary conditions.

The scientists and engineers at present have serious concern about enhancement for the rate of heating/ cooling and energy storage in numerous modern and progressive technologies. Many attempts have been made in this direction by considering different types of wall to ambient temperature distributions using different fluid models. Still main problem of the scientist is for the high storage of energy with low cost. Some progressive technologies such as solar energy, waste heat recovery and combined heat and power plants need economically more suitable and efficient way to store the energy. Generally three methods are used for energy storage i.e., latent heat energy, sensible heat energy and chemical thermal energy storage. Among these latent heat energy is considered economically more suitable and efficient (by changing the phase of materials). Therefore thermal energy is stored in a material through a latent heat by melting process. Melting aspect is very essential in processes including melting of soil, the freezing of soil around the heat exchanger coils of a ground based pump, melting of permafrost, magma solidification, thawing of frozen grounds, the freeze treatment of sewage, the preparation of semiconductor material, the casting and welding of a manufacturing process [7]. Hayat et al. [8] considered stagnation point flow of Powell–Eyring fluid towards a linear stretching surface in the presence of melting phenomenon. Characteristics of melting heat transfer in the stagnation point flow of Burgers fluid is reported by Awais et al. [9]. Behavior of melting heat transfer in the magnetohydrodynamic (MHD) flow over a moving surface is investigated by Das [10]. Yacob et al. [11] analyzed the characteristics of melting phenomenon in boundary layer stagnation point flow of

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micropolar liquid over a stretching/shrinking surface. Hayat et al. [12] studied the boundary layer stagnation point flow of third grade fluid towards a stretching surface with melting effects. Bachok et al. [13] explored the boundary layer stagnation point flow of viscous fluid over a stretching sheet in the presence of melting heat transfer. Effects of melting process in the stagnation point flow of Maxwell fluid with double diffusive effect are investigated by Hayat et al. [14]. Mustafa et al. [15] examined the characteristics of melting heat transfer in the flow of Jeffery fluid in the presence of viscous dissipation. Awais et al. [16] studied the stagnation point flow with melting phenomena in the presence of thermal diffusion and diffusion thermo effects.

The main motto of the present paper is to address the high rate of heating/cooling of fluids and high storage of energy by latent heat. Hence melting heat transfer in magnetohydrodynamic stagnation point flow of Tangent hyperbolic fluid towards a vertical surface is discussed. Mathematical formulation for relevant problems is developed using boundary layer theory. Ambient temperature is assumed higher than the melting surface temperature. Convergent series solutions of the resulting nonlinear problems are computed using the homotopy analysis method [17–38]. The velocity, temperature, skin friction coefficient and Nusselt number are analyzed for different emerging parameters. Comparison with previous studies [39,40] in limiting cases is also given.

2. Mathematical formulation

Consider the steady mixed convection flow of tangent hyperbolic electrically conducting fluid due to a vertically stretched surface. The x-axis is taken along the surface in vertical direction and y-axis normal to the plane of the plate. The geometry of flow problem is presented in Fig. 1. The fluid is permeated by uniform applied magnetic field B_0 in a direction making an angle ψ with the positive direction of x-axis. Magnetic Reynolds number is taken small and induced magnetic field thus is neglected. No electric field is considered and applied magnetic field $\mathbf{B} \equiv (B_0 \cos\psi, B_0 \sin\psi, 0)$. Heat transfer is studied through melting heat mechanism. Taking into account the aforementioned assumptions, the governing boundary layer equations are represented as follows:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{1}$$

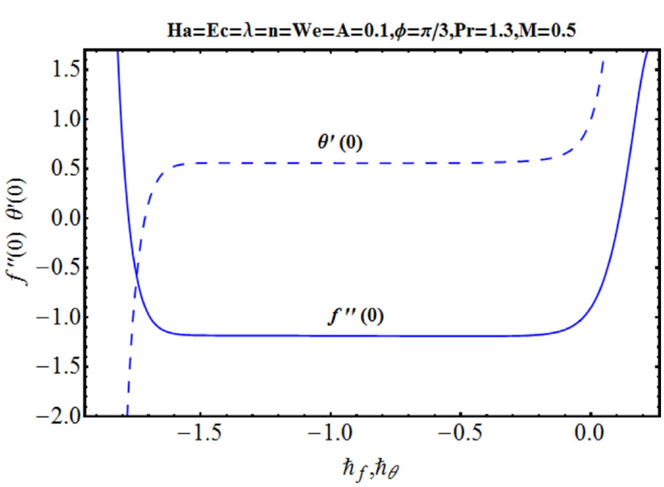

Fig. 1. $\hbar$–curves for the functions $f(\eta)$ and $\theta(\eta)$.

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = U_e\frac{dU_e}{dx} + \nu(1-n)\frac{\partial^2 u}{\partial y^2} + 2\nu\Gamma n\frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial y^2} + \frac{\sigma B_0^2 \sin^2\phi}{\rho}(U_e - u) + g\beta_T(T - T_m), \tag{2}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{k}{\rho c_p}\frac{\partial^2 T}{\partial y^2} + \frac{\nu}{c_p}(1-n)\left(\frac{\partial u}{\partial y}\right)^2 + \frac{\nu}{c_p}n\Gamma\left(\frac{\partial u}{\partial y}\right)^3 + \frac{\sigma B_0^2 \sin^2\phi}{\rho c_p}(u - U_e)^2. \tag{3}$$

The boundary conditions are prescribed in the following forms:

$$u(x,0) = U_w = U_0 \exp\left[\frac{x}{l}\right], \quad T(x,0) = T_m, \tag{4}$$

$$u \to U_e = U_\infty \exp\left[\frac{x}{l}\right], \quad T \to T_\infty, \quad \text{as } y \to \infty. \tag{5}$$

and

$$k\left(\frac{\partial T}{\partial y}\right)_{y=0} = \rho\left[\beta + c_s(T_m - T_0)\right]v(x,0). \tag{6}$$

Here u and v are the velocity components in the x and y directions respectively, ρ the fluid density, σ the electrical conductivity of fluid, k the thermal conductivity, g the acceleration due to gravity, β_T the volumetric coefficient of thermal expansion, T the temperature of fluid, c_p the specific heat, ν the kinematic viscosity, $U_w = U_0 \exp[\frac{x}{L}]$ the stretching velocity, $U_e = U_\infty \exp[\frac{x}{L}]$ the free stream velocity, U_0 the reference velocity, β the latent heat of the fluid, c_s the heat capacity of the solid surface and T_m the melting surface temperature. The boundary condition in Eq. (6) shows that the heat conducted to the melting surface is equal to the heat of melting plus the sensible heat required to raise the solid temperature T_0 to its melting temperature T_m.

The first term on the R.H.S of Eq. (2) is due to the stagnation point flow while second and third terms are because of Tangent hyperbolic fluid. The fourth term in the R.H.S. of Eq. (2) presents the effect of magnetic force with the angle of inclination (ψ) while the fifth term is due to mixed convection. The second and third terms in the R.H.S. of Eq. (3) are for the viscous dissipation and the fourth term corresponds to Joule heating.

Using the transformations [33]:

$$\eta = \sqrt{\frac{U_0}{2\nu l}}\exp\left(\frac{x}{2l}\right)y, \quad \psi = \sqrt{2\nu l U_0}f(\eta)\exp\left(\frac{x}{2l}\right)$$
$$u(x,y) = U_0\exp\left(\frac{x}{l}\right)f'(\eta),$$
$$v(x,y) = -\sqrt{\frac{\nu U_0}{2l}}\exp\left(\frac{x}{2l}\right)\left[f(\eta) + \eta f'(\eta)\right],$$
$$\theta = \frac{T - T_m}{T_\infty - T_m}, \tag{7}$$

incompressibility condition is identically satisfied and Eqs. (2)–(6) are reduced into the following problems:

$$(1-n)f''' - 2f'^2 + ff'' + nWef''f''' + A^2 + (Ha)^2\sin^2\phi(A - f') + 2\lambda\theta = 0, \tag{8}$$