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Detection of ferromagnetic target based on mobile magnetic gradient tensor system

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ABSTRACT

Attitude change of mobile magnetic gradient tensor system critically affects the precision of gradient measurements, thereby increasing ambiguity in target detection. This paper presents a rotational invariant-based method for locating and identifying ferromagnetic targets. Firstly, unit magnetic moment vector was derived based on the geometrical invariant, such that the intermediate eigenvector of the magnetic gradient tensor is perpendicular to the magnetic moment vector and the source–sensor displacement vector. Secondly, unit source–sensor displacement vector was derived based on the characteristic that the angle between magnetic moment vector and source–sensor displacement is a rotational invariant. By introducing a displacement vector between two measurement points, the magnetic moment vector and the source–sensor displacement vector were theoretically derived. To resolve the problem of measurement noises existing in the realistic detection applications, linear equations were formulated using invariants corresponding to several distinct measurement points and least square solution of magnetic moment vector and source–sensor displacement vector were obtained. Results of simulation and principal verification experiment showed the correctness of the analytical method, along with the practicability of the least square method.

1. Introduction

Detection technology utilizing local magnetic anomalies of ferromagnetic objects has become an important trend in target detection [1]. Localization and magnetic moment estimation is conventionally realized by measuring the total magnetic intensity or magnetic field vectors, using magnetometers installed on a mobile platform. The target detection technique based on magnetic anomaly has preferable imperceptibility in contrast to some active detection methods, which has led to its widespread exaltation.

Along with development of the theory as well as techniques for the magnetic anomaly detection, several magnetic gradient tensor systems comprising multiple magnetometers have been developed over recent years. Magnetic gradient tensor measurements have many overwhelming majorities over conventional magnetic scalar and vector probes [2]. Gradient measurements mainly reflect gradients from anomalous sources because the background geomagnetic gradient is low. Simultaneously, gradient measurements can also provide additional valuable information to assist interpretation of anomalous sources compared to the field-magnitude and field-component measurements [3].

Wynn et al. [4–6] have carried out extensive and innovative work on localization of magnetic dipole based on magnetic gradient tensor data. In recent years, a set of linear equations that relate dipole location to magnetic field vector and magnetic gradient tensor at a single measurement point was proposed for magnetic dipole localization [3,7,8]. This set of equations is equivalent to the Euler's equations, which are widely used in geophysics. However, this method lacks the precision in measurement of magnetic field-components, which relates dipole position and the measurement point. In a real application, the measurement data is magnetic field vector consisting of the background geomagnetic field and the magnetic field generated by magnetic sources. The magnetic anomaly field of ferromagnetic materials is highly indistinguishable from the background geomagnetic field, leading to large errors in the estimated position. On the other hand, errors in the measured orientation of the magnetic field vectors also affect localization precision for the mobile measurement platform. Later on, a magnetic gradient tensor system comprising an array of eight tri-axial fluxgate magnetometers was designed and the Scalar Triangulation And Ranging (STAR) method based on magnetic gradient tensor invariants was proposed for achieving magnetic dipole localization [9]. Though this method overcomes the aforementioned shortcoming, it also poses

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calibration difficulties for the system. A few nonlinear numerical estimation methods consisting of Bayesian [10] and inversion algorithms [11] were also proposed to comprehend the magnetic gradient tensor data based ferromagnetic object localization approach. But the convergence rates and localization instantaneity still requires further improvement.

Certain combinations of magnetic gradient tensor components are independent with reference frame, and they are invariant with the posed variation of magnetic gradient tensor system. Hereinto, eigenvalues of magnetic gradient tensor matrix are important invariants. Majid Beiki [12,13] used eigenvector analysis of gravity gradient tensor and pseudogravity gradient tensor to locate geologic bodies, where eigenvectors corresponding to the smallest eigenvalue and the maximum eigenvalue were used. According to Beiki et al. [14], it is inferred that eigenvector corresponding to the intermediate eigenvalue is perpendicular to the dipole moment vector and source–sensor displacement vector. This special geometrical relationship motivates toward its utilization in the detection of magnetic sources by a mobile magnetic gradient tensor system. Hence, a novel target detection method is proposed in this paper, based on the geometrical and mathematical relationship among eigenvector, source–sensor displacement vector and dipole moment. Location and magnetic moment are estimated using the intercepted continuous data acquired from the magnetic gradient tensor system.

This paper is organized as follows: in Section 2 we present measurement principle and rotational invariants of magnetic gradient tensor; in Section 3 we introduce the analytic solution of location and magnetic moment using eigenvectors of magnetic gradient tensor; in Section 4, a least square method is proposed for taking into account measurement noises. In Section 5 we test the performance of the algorithm by running simulations, which are followed by the real experiments in Section 6. Finally, Section 7 concludes the findings of this paper.

2. Magnetic gradient tensor measurement principle and rotational invariants

2.1. Magnetic gradient tensor measurement principle

Magnetic gradient tensor is the spatial rate of change of magnetic field vector in three orthogonal directions. $\boldsymbol{B}$ is described as magnetic field vector, with the magnetic gradient tensor $\boldsymbol{G}$ expressed shown as multiplication of two matrices containing the three vector components.

$$\boldsymbol{G}=\begin{bmatrix}\partial/\partial x\\ \partial/\partial y\\ \partial/\partial z\end{bmatrix}\begin{bmatrix}\boldsymbol{Bx} & \boldsymbol{By} & \boldsymbol{Bz}\end{bmatrix}=\begin{bmatrix}B_{xx} & B_{xy} & B_{xz}\\ B_{yx} & B_{yy} & B_{yz}\\ B_{zx} & B_{zy} & B_{zz}\end{bmatrix} \tag{1}$$

where $\boldsymbol{Bx}$, $\boldsymbol{By}$ and $\boldsymbol{Bz}$ are measured magnetic field components in three orthogonal directions, $B_{ij}, i, j = x, y, z$ denote tensor components.

The geomagnetic field and magnetic anomaly caused by the ferromagnetic matter are magnetostatic fields which do not contain conduction currents. So the curl and divergence of the magnetic field vanishes according to Maxwell's magnetostatic equations.

$$\begin{cases}\nabla\bullet\boldsymbol{B}=\dfrac{\partial\boldsymbol{Bx}}{\partial x}+\dfrac{\partial\boldsymbol{By}}{\partial y}+\dfrac{\partial\boldsymbol{Bz}}{\partial z}=0\\ \nabla\times\boldsymbol{B}=0\end{cases} \tag{2}$$

According to Eqs. (1) and (2), $\boldsymbol{G}$ is symmetric and only five of the nine tensor components are independent.

2.2. Rotational invariants of magnetic gradient tensor

Magnetic gradient tensor components are relative to the measurement reference frame. However, combinations of certain components are independent of the choice of the measurement frame of reference, and they are the rotational invariants of the magnetic gradient tensor. $\boldsymbol{G}$ is a symmetric traceless 3×3 matrix, it can be formulated into an eigenvalue problem as

$$\boldsymbol{G}=\begin{bmatrix}\boldsymbol{b}_1 & \boldsymbol{b}_2 & \boldsymbol{b}_3\end{bmatrix}^T\begin{bmatrix}\lambda_1 & & \\ & \lambda_2 & \\ & & \lambda_3\end{bmatrix}\begin{bmatrix}\boldsymbol{b}_1 & \boldsymbol{b}_2 & \boldsymbol{b}_3\end{bmatrix} \tag{3}$$

where λ_1, λ_2 and λ_3 are eigenvalues and $\boldsymbol{b}_1$, $\boldsymbol{b}_2$, $\boldsymbol{b}_3$ are the corresponding mutually orthogonal eigenvectors. The eigenvalues are actually the rotational invariants of magnetic gradient tensor and the other invariants can be expressed in terms of these eigenvalues.

The eigenvalues can be found by solving the following characteristic equation.

$$\det(\lambda\boldsymbol{I}-\boldsymbol{G})=\lambda^3-I_0\lambda^2+I_1\lambda-I_2=0 \tag{4}$$

where I_0, I_1 and I_2 are the invariants. They are combinations of some of the magnetic gradient tensor components:

$$\begin{cases}I_0=B_{xx}+B_{yy}+B_{zz}=0\\ I_1=B_{xx}B_{yy}+B_{yy}B_{zz}+B_{xx}B_{zz}-B_{xy}^2-B_{yz}^2-B_{xz}^2\\ I_2=B_{xx}(B_{yy}B_{zz}-B_{yz}^2)+B_{xy}(B_{yz}B_{xz}-B_{xy}B_{zz})\\ \quad+B_{xz}(B_{xy}B_{yz}-B_{xz}B_{yy})\end{cases} \tag{5}$$

According to the Eq. (4), eigenvalues of magnetic gradient tensor can be expressed as

$$\begin{cases}\lambda_1=C+D\\ \lambda_2=-\dfrac{C+D}{2}+\dfrac{C-D}{2}\sqrt{-3}\\ \lambda_3=-\dfrac{C+D}{2}-\dfrac{C-D}{2}\sqrt{-3}\end{cases} \tag{6}$$

where $C=\{I_2/2+[(I_2/2)^2+(I_1/3)^3]^{1/2}\}^{1/3}$, $D=\{I_2/2-[(I_2/2)^2+(I_1/3)^3]^{1/2}\}^{1/3}$, and $\lambda_1>\lambda_2>\lambda_3, \lambda_1>0>\lambda_3$, $|\lambda_2|<|\lambda_3|$, $|\lambda_2|<|\lambda_1|$. $\lambda_2$is the intermediate eigenvalue that has the smallest absolute value.

3. Analytical methods based on magnetic gradient tensor invariants for target inversion

Rotational invariants of magnetic gradient tensor remain constant while the measurement system rotates around a fixed point. This characteristic has been widely employed in environmental, military, and medical applications [15–16]. Eigenvector analysis of tensor field has also been used in location and edge detection of geological bodies [12,17], as well as in interpreting aeromagnetic data [13]. Beiki et al. [14] proposed the geometrical and mathematical relationship among the source location, the corresponding dipole moment and the eigenvectors corresponding to the intermediate eigenvalue. However, they only used the normalized source strength, derived from eigenvalues, to interpret magnetic gradient tensor data, and did not explicitly use the geometrical relationship of eigenvectors. Due to fact that the geometrical relationship does not vary under coordinate changes, it is used to demonstrate estimation of location and magnetic moment of magnetic sources in this paper, along with the derivation of the corresponding analytical solution.