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Stability of topological charge of magnetic skyrmion configurations

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ABSTRACT

We analyze the topological charge of a skyrmion q_s, and the corresponding Hall conductivity σ_{xy}, which can serve as an electrical read-out for skyrmion-based memory. We derived the general form of the Dzyaloshinskii–Moriya (DM) interaction for any arbitrary orientation of the DM vector $\boldsymbol{D}$. Based on the DM interaction energy, we obtained the dependence the skyrmion helicity angle γ on the orientation of $\boldsymbol{D}$. We showed via general mathematical arguments, the topological nature of the skyrmionic charge q_s, and its independence of γ and specific details of the interior of the skyrmion (e.g., its core size). Finally, we showed via numerical micromagnetics the stability of q_s under varying applied B-fields till the annihilation field, despite the drastic reduction in the skyrmion core size.

Skyrmions, originally introduced as a model for hadrons in nuclear physics [1], are topological objects [2,3] associated with integer invariants, known as their topological charge. Since this invariant is quantized, it is not subject to continuous change due to smooth deformations in the system, thus conferring a high degree of stability to the skyrmion. Skyrmion-like configurations have been observed in magnetic materials, where there are competing interaction terms which result in the canting of the magnetization. In magnetic thin films, the competition between magnetic anisotropy in the perpendicular direction and magnetostatic energy which favours in-plane configuration can lead to the formation of the so-called giant skyrmions [4] of a few hundred microns in size. More recently, smaller magnetic skyrmions of a few tens of nanometers have been observed in various non-centrosymmetric magnets which exhibit the Dzyaloshinskii–Moriya (DM) interaction, which is a type of super-exchange mediated by ions with spin–orbit interaction [5,6]. Such DM-induced skyrmions have been reported in helimagnets such as MnSi [7,8], FeGe [9,10] and $Fe_{1-x}Co_xSi$ [11], and in multiferroic insulators such as Cu_2OSeO_3 [12]. In view of their compact size and robustness conferred by their topological property [13–15], skyrmions have elicited much interest as possible candidates for high-density memory elements. Furthermore, it has been shown that current-induced motion of skyrmions can be induced by current densities as low as $\sim 10^5\ \mathrm{A/m^5}$ [16,17], while controlled writing and annihilation of individual skyrmions via spin injection from a scanning tunneling microscope has also been demonstrated on PdFe bilayer on Ir(111) [18].

Although the presence of skyrmions can be characterized by neutron scattering [8] and Lorentz microscopy [9], for memory application, it would be useful to have an electrical read-out method, especially one which is sensitive to the skyrmion's topological charge q_s. In this paper, we investigate the electrical read-out based on the skyrmionic charge. First, we analyzed the general DM interaction energy of a skyrmion for any arbitrary orientation of the DM vector $\boldsymbol{D}$. Based on the expression for the DM interaction energy and assuming a skyrmion ring model, we obtained the dependence the skyrmion helicity angle γ on the orientation of $\boldsymbol{D}$. Subsequently, we showed by general mathematical arguments, the topological nature of the skyrmionic charge q_s and its independence of the helicity in any parameter space. The topological nature of q_s suggests the stability of any property which is dependent on q_s, such as the Hall conductivity σ_{xy}. We performed exemplary micromagnetic simulation to show the robustness of q_s under a perpendicular B-field, and hence the suitability of utilizing the Hall conductivity σ_{xy} for electrical read-out of skyrmion-based memory.

1. DM energy of skyrmions

First, we consider the different forms of the micromagnetic energy density due to the DM interaction within a two-dimensional magnetic skyrmion configuration as the orientation of the DM vector $\boldsymbol{D}$ varies with respect to the plane of the skyrmion as well as the separation vector between neighbouring moments. If we denote the spatial texture of the skyrmion by $\boldsymbol{n}(\boldsymbol{r})$, then its the

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free energy density ε can be written as [19]

$$\varepsilon = \varepsilon_{ex} + \varepsilon_a + \varepsilon_Z + \varepsilon_{DM} = A(\nabla \boldsymbol{n})^2 + K_u(\boldsymbol{n}\cdot\boldsymbol{w}_a)^2 + M_s(\boldsymbol{n}\cdot\boldsymbol{H}_a) + \varepsilon_{DM}, \quad (1)$$

where the first three terms correspond to the usual free energy terms of standard micromagnetics in the continuum limit (namely, the exchange, magnetocrystalline anisotropy and Zeeman energies) [20,21]. Here, A is the exchange stiffness, K_u is the anisotropy constant, $\boldsymbol{w}_a$ is the easy axis direction, $\boldsymbol{H}_a$ is the applied magnetic field and ε_{DM} is the DM interaction energy. The Hamiltonian of the system is then given by the volume integral of ε [2].

To evaluate the form of ε_{DM}, we note that the DM interaction causes canting between neighbouring moments and thus, the discrete DM energy between nearest neighbours i and j can be expressed as $\varepsilon_{DM}^{ij} = \boldsymbol{D}\cdot(\boldsymbol{n}_i \times \boldsymbol{n}_j)$, where the DM vector $\boldsymbol{D}$ sets the axis of spin canting. In terms of spatial derivatives, one can express the interaction energy as

$$\varepsilon_{DM}^{ij} = \boldsymbol{D}\cdot(\boldsymbol{n}_i \times \Delta\boldsymbol{n}_{ij}) = \boldsymbol{D}\cdot[\boldsymbol{n}(\boldsymbol{r}_i) \times (\boldsymbol{r}_{ij}\cdot\nabla)\boldsymbol{n}(\boldsymbol{r}_i)], \quad (2)$$

where $\boldsymbol{r}_{ij}$ is the separation vector. In the case where $\boldsymbol{D}\|\hat{\boldsymbol{r}}_{ij}$, i.e., $\boldsymbol{D} = D_\|\hat{\boldsymbol{r}}_{ij}$, and considering a 2D square lattice with nearest neighbours at $\hat{\boldsymbol{r}}_{ij} = \{\pm a\hat{\boldsymbol{x}}, \pm a\hat{\boldsymbol{y}}\}$, we have

$$\begin{aligned}\varepsilon_{DM}^{\|}(\boldsymbol{r}_i) &\approx D_\| a\Big[\hat{\boldsymbol{x}}\cdot\Big(\boldsymbol{n}\times\partial_x\boldsymbol{n}\Big) + \hat{\boldsymbol{y}}\cdot\Big(\boldsymbol{n}\times\partial_y\boldsymbol{n}\Big)\Big] \\ &= D_\| a(\epsilon_{xqr}n_q\partial_x n_r + \epsilon_{yqr}n_q\partial_y n_r + \epsilon_{zqr}n_q\partial_z n_r) \\ &= D_\| a\epsilon_{pqr}n_q\partial_p n_r = -D_\| a\epsilon_{pqr}n_p\partial_q n_r \\ &= -D_\|'\boldsymbol{n}(\boldsymbol{r}_i)\cdot(\nabla\times\boldsymbol{n}(\boldsymbol{r}_i)),\end{aligned} \quad (3)$$

where in the second line of the above we have added the null (third) term, and $D_\|' = D_\| a$. The approximation in the first line of Eq. (3) holds if the spin texture varies over a length-scale much larger than a. The form of ε_{DM} in Eq. (3) is useful in the continuum limit and is equivalent to the form given in Refs. [22] and [2]. Likewise, we can derive the continuum expression in the case where $\boldsymbol{D}$ is perpendicular to $\boldsymbol{r}_{ij}$ but lying on the plane of atoms, i.e., $\boldsymbol{D}\|(\boldsymbol{r}_{ij}\times\hat{\boldsymbol{z}})$. Summing over the four neighbours as before, we have

$$\begin{aligned}\varepsilon_{DM}^{\perp} &\approx D_\perp a\Big[(\hat{\boldsymbol{x}}\times\hat{\boldsymbol{z}})\cdot\Big(\boldsymbol{n}\times\partial_x\boldsymbol{n}\Big) + (\hat{\boldsymbol{y}}\times\hat{\boldsymbol{z}})\cdot\Big(\boldsymbol{n}\times\partial_y\boldsymbol{n}\Big)\Big] \\ &= D_\perp a\hat{\boldsymbol{z}}\cdot\Big[(\boldsymbol{n}\times\partial_i\boldsymbol{n})\times\hat{\boldsymbol{x}}_i\Big] \equiv D_\perp a\hat{\boldsymbol{z}}\cdot\boldsymbol{v},\end{aligned} \quad (4)$$

where $v_j = \varepsilon_{jkl}(\boldsymbol{n}\times\partial_l\boldsymbol{n})_k$. This may be simplified to

$$\begin{aligned}v_j &= \varepsilon_{jkl}(\boldsymbol{n}\times\partial_l\boldsymbol{n})_k = \varepsilon_{jkl}\varepsilon_{kmn}n_m\partial_l n_n = \big(\delta_{lm}\delta_{jn} - \delta_{ln}\delta_{jm}\big)(n_m\partial_l n_n) \\ &= n_l\partial_l n_j - n_j\partial_l n_l\end{aligned}$$

$$\Rightarrow \boldsymbol{v} = [(\boldsymbol{n}\cdot\nabla)\boldsymbol{n}] - \boldsymbol{n}(\nabla\cdot\boldsymbol{n}), \quad (5)$$

and so the DM energy in continuum form for the case of perpendicular DM vector is given by

$$\varepsilon_{DM}^{\perp} = D_\perp'[(\boldsymbol{n}\cdot\nabla)n_z] - n_z(\nabla\cdot\boldsymbol{n}), \quad (6)$$

where $D_\perp' = D_\perp a$.

2. Helicity of skyrmions

The helicity of a skyrmion describes the clockwise or anticlockwise twist of the magnetization orientation within the skyrmion about its core, and is characterized by the helicity angle γ [2,23,24]. Based on the DM energetics discussed in the previous section, we can derive the equilibrium skyrmion configuration and, hence its helicity under varying orientation of the DM vector $\boldsymbol{D}$. We first express the unit vector $\boldsymbol{n}$ along the magnetization direction (which denotes the spin texture of the skyrmion) in terms of the polar and azimuthal spin angles, i.e.,

$$\boldsymbol{n} = (\sin\theta(\boldsymbol{r})\cos\phi(\boldsymbol{r}), \sin\theta(\boldsymbol{r})\sin\phi(\boldsymbol{r}), \cos\theta(\boldsymbol{r})). \quad (7)$$

Due to the rotational symmetry of a general skyrmion configuration, the spin angles θ and ϕ are functions of r and φ only, i.e., $\theta(\boldsymbol{r}) = \theta(r)$ and $\phi(\boldsymbol{r}) = \phi(\varphi)$, where (r, φ) are the spatial cylindrical coordinates. In addition, the polar angle can be any smooth function which obeys the following boundary conditions:

$$\theta(r) = \begin{cases}(P-1)\dfrac{\pi}{2}, & r = 0,\\ (P+1)\dfrac{\pi}{2}, & r\to\infty,\end{cases} \quad (8)$$

where $P = \pm 1$ is the polarity, while the general form for the azimuthal spin angle is $\phi(\varphi) = W\varphi + \gamma$, where W is an integer signifying the winding number, and γ is the helicity angle. In this paper, we will limit the discussion to skyrmions of unit winding number. For the special helicity angles of $\gamma=0$ and $\gamma = \pi/2$, the skyrmion adopts the (outward) radial and (counter-clockwise) vortex forms, respectively, which can be mapped to the regular *hedgehog* or *combed hedgehog* on the unit Bloch sphere in spin space [25].

We consider the DM interaction energy between a moment at the centre of a skyrmion and a circular ring of neighbouring moments situated at δr away in the radial direction. For simplicity, we consider polarity $P=1$, i.e., the polar angle at the centre is $\theta = 0$, while that of the neighbours is $\delta\theta = (\partial_r\theta)\delta r$. Therefore, the change in the moment orientation over $(\delta r)\hat{\boldsymbol{r}}$ is $\Delta\boldsymbol{n} = (\cos\phi, \sin\phi, 0)(\partial_r\theta)\delta r$. Now, suppose the DM vector is aligned within the plane of the moments but at some angle ϑ to the separation direction $\hat{\boldsymbol{r}}$, i.e., $\boldsymbol{D} = D(\cos(\varphi+\vartheta), \sin(\varphi+\vartheta), 0)$. Thus, following Eq. (2) and summing over the ring of neighbours, the total DM interaction energy is given by

$$\begin{aligned}\varepsilon_{DM} \sim \int\boldsymbol{D}\cdot(\boldsymbol{n}\times\Delta\boldsymbol{n})\,d\varphi &= D(\partial_r\theta)\delta r\int_0^{2\pi}\sin(\varphi+\vartheta-\phi(\varphi))\,d\varphi \\ &= -D(\partial_r\theta)\delta r\int_0^{2\pi}\sin[(W-1)\varphi+\gamma \\ &\quad -\vartheta]\,d\varphi.\end{aligned} \quad (9)$$

Supposing $D > 0$ and $W=1$, the DM energy is thus minimized when $\gamma = \vartheta + \pi/2$. Thus, when $\boldsymbol{D}\|\hat{\boldsymbol{r}}$, i.e., $\vartheta = 0$, we have $\gamma = \pi/2$, which corresponds to the counter-clockwise vortex skyrmion. Conversely, for $\boldsymbol{D}\perp\hat{\boldsymbol{r}}$, i.e., $\vartheta = \pi/2$, $\gamma = \pi$, which corresponds to the inward radial skyrmion. For $D < 0$, the direction and sense of rotation of the skyrmions are reversed. Finally, if $\boldsymbol{D}$ is at some oblique angle to $\hat{\boldsymbol{r}}$, then the resulting skyrmion would have intermediate degree of helicity with $0 < \gamma < \pi/2$. The dependence of the skyrmion helicity to the direction of $\boldsymbol{D}$ is depicted in Fig. 1.

3. Skyrmionic charge and Hall conductivity

It has been shown both theoretically [15,26] and experimentally [27,28] that an electron which traverses through a 2D skyrmionic texture (on the xy plane) and relaxes to the local magnetization direction $\boldsymbol{n}(\boldsymbol{r})$ would experience an *emergent* out-of-plane magnetic field:

$$B_t^z = \frac{\Phi_0}{S}\int\boldsymbol{n}\cdot\big(\partial_x\boldsymbol{n}\times\partial_y\boldsymbol{n}\big)\,dx\,dy \equiv 4\pi\Big(\frac{\Phi_0}{S}\Big)q_s, \quad (10)$$

where $\Phi_0 = \hbar/2e$ is the flux quantum, S is the area of the skyrmion and q_s is the skyrmionic charge. The emergent field causes a transverse deflection of electrons, resulting in an additional contribution to the Hall conductivity σ_{xy} over and above the normal Hall (due to applied B-field) and anomalous Hall (due to the net