Journal of Magnetism and Magnetic Materials 398 (2016) 253–258

Contents lists available at ScienceDirect

Journal of Magnetism and Magnetic Materials

journal homepage: www.elsevier.com/locate/jmmm

The van Hemmen model and effect of random crystalline anisotropy field

Denes M. de Morais [a], Mauricio Godoy [a,*], Alberto S. de Arruda [a], Jonathas N. da Silva [b], J. Ricardo de Sousa [c]

[a] Instituto de Física, Universidade Federal de Mato Grosso, 78060-900 Cuiabá, Mato Grosso, Brazil
[b] Universidade Estadual Paulista, 14800-901, Araraquara, São Paulo, Brazil
[c] Instituto Nacional de Sistemas Complexos, Departamento de Fisica, Universidade Federal do Amazona, 69077-000, Manaus, Amazonas, Brazil

ARTICLE INFO

Article history:
Received 29 June 2015
Accepted 10 September 2015
Available online 12 September 2015

Keywords:
Random crystalline field
Van Hemmen model
Phase diagram

ABSTRACT

In this work, we have presented the generalized phase diagrams of the van Hemmen model for spin $S=1$ in the presence of an anisotropic term of random crystalline field. In order to study the critical behavior of the phase transitions, we employed a mean-field Curie–Weiss approach, which allows calculation of the free energy and the equations of state of the model. The phase diagrams obtained here displayed tricritical behavior, with second-order phase transition lines separated from the first-order phase transition lines by a tricritical point.

1. Introduction

The interest in understanding the physical properties of disordered systems provides a fertile ground of novel models and studies in statistical physics. Many disordered systems are known to be of fundamental importance, including the so-called spin-glass (SG) [1–3] and the random field Ising model (RFIM). The RFIM is a topic of interest current and one of the reasons for this is its physical realization as a diluted Ising antiferromagnet in the presence of a uniform magnetic field along the uniaxial direction [4]. Several of its properties have been revealed, particularly the lower critical ($d_l=2$) dimension above which a stable ferromagnetic state exists at low temperatures [5]. The phase diagram is another interesting feature of RFIM. Here, the use of the mean-field theory has shown that the nature of the phase transition depends on the distribution associated with the random magnetic field. For Gaussian distribution, the phase transition is always continuous [6], and with a symmetric bimodal distribution the phase transition is continuous (second-order) in the region of low and high temperatures, becoming discontinuous (first-order) for sufficiently large values of magnetic field (random) and low temperature [7]. The trimodal distribution [8–11] indicates a critical p_c for which the tricritical behavior disappears. Other studies have indicated that in the three-dimensional case a jump in magnetization and no latent heat is observed [12,13]. Moreover, also very important systems are the spin glasses [14]. Research in spin glasses began in the 1970s after the discoveries of a peak in AC susceptibility in dilute alloys of gold and iron [15]. Basically, the spin glasses are dilute magnetic alloys, where the interactions between spins are randomly ferromagnetic or antiferromagnetic distributed, which are examples of frozen disorder. This is a necessary requirement for having a locally random competition between ferromagnetic and antiferromagnetic interactions. The presence of disorder introduces frustration and produces difficulties for the system to form an optimized configuration. These heterogeneities of the couplings lead to frustrated spin arrangements, that is, for any choice of orientations of spins leaves we have at least one frustrated bond. There are a large number of examples of spin glass systems, which include CuMn and AuFe alloys [16]. These systems are known to exhibit non-trivial thermodynamic and dynamic properties, which are different when compared to those observed in ordered systems.

Ising-type Hamiltonians with random bonds between the spins are used to spin glass models. In mean-field models the effects of frustration are enhanced, and the thermodynamic that emerges is in some case novel and surprising. On one hand, when compared to ordinary ferromagnets, which host two pure states present below of the transition temperature, the mean-field spin glasses model exhibit at low temperature a complex structure of infinitely large number of equilibrium states, organized in a hierarchical structure. Formally, there is a mathematical approach that enables us to appropriately deal with the quenched disorder present in the Hamiltonian which allows us to compute the thermodynamic properties, but this is very difficult problem to be treated.

Recently, the Ising spin glass with random field similar to model proposed by Sherrington and Kirkpatrick (SK) [17] has been treated for Gaussian [18] and bimodal [19] distributions. The mean-field spin glass model has often been employed in the study of the dynamic anomalies, which are associated with relaxational phenomena in

* Corresponding author.
E-mail addresses: mgodoy@fisica.ufmt.br (M. Godoy), aarruda@fisica.ufmt.br (A.S. de Arruda).

http://dx.doi.org/10.1016/j.jmmm.2015.09.050
0304-8853/

supercooled liquids and real glasses [20]. The aging and breakdown of the fluctuation dissipation theorem are intrinsic features of these systems that are captured by dynamic versions of such models. The dynamic calculations are frequently used, but often solutions are found only numerically [21].

It is worth noting that the problems related to spin glasses remain difficult to treat in statistical mechanics [22–24]. Up to now, only mean-field models are exactly tractable, but they require sophisticated mathematical tools [25,26]. Therefore, disorder in spin systems is a permanent source of challenging problems. The spin-glass (SG) state is one of the most interesting examples where disorder can provide new physics properties and responses.

In this present work, we have studied the influence of the random crystalline anisotropy field on the spin glass mean field model introduced by van Hemmen [27–29]. This model is exactly solvable and unlike the Sherrington–Kirkpatrick model [17], its solution does not require the use of replica trick. In spite of being not fully realistic, mean-field models give a first qualitative understanding of the thermodynamic behavior of a system.

The outline of the paper is as follows: in Section 2, we have introduced the model, where obtained an analytical expression for the free-energy and equation of states. In Section 3, we presented and discussed the phase diagrams. Finally, Section 4 is devoted to our conclusions.

2. Formalism and calculation

The Hamiltonian of the van Hemmen spin-glass model, with an additional ferromagnetic Ising term and a random crystal field, is given by

$$\mathcal{H} = -\frac{J_0}{N}\sum_{\langle i,j\rangle} S_iS_j - \sum_{\langle i,j\rangle} J_{ij}S_iS_j - H\sum_i S_i - \sum_i D_iS_i^2, \tag{2.1}$$

where $J_0 > 0$ is the direct ferromagnetic coupling between the spins in the nearest-neighbor sites $i$and j, which is fundamental to the existence of a mixed phase. H is an external uniform magnetic field and J_{ij} are random interactions between the nearest-neighbor spins, which is given by

$$J_{ij} = \frac{J}{N}\left(\xi_i\eta_j + \xi_j\eta_i\right). \tag{2.2}$$

Here, ξ_i and η_j are independent random variables, obeying the probability distribution $P(\xi_i, \eta_j)$

$$P(\xi,\eta) = \left\{\frac{1}{2}\left[\delta(\xi_i - 1) + \delta(\xi_i + 1)\right]\right\}\left\{\frac{1}{2}\left[\delta(\eta_j - 1) + \delta(\eta_j + 1)\right]\right\}, \tag{2.3}$$

which is expected to mimic the presence of disorder in real spin glasses. Moreover, in Eq. (2.1) $\langle i, j\rangle$ denote the sum over all nearest-neighbor pairs of spins on a d-dimensional lattice. In that follows, we have considered $S_i > 1/2$ (due to the presence of D_i) which are spin variables integer or half-integer. The crystalline anisotropy fields (or crystal fields) D_i are subject to a bimodal probability distribution

$$P(D_j) = p\delta(D_j) + (1-p)\delta(D_j - D), \tag{2.4}$$

where the first term $p\delta(D_j)$ indicates that a fraction p of the spins are free of the influence of crystalline anisotropy field, and in the second term $(1-p)\delta(D_j - D)$ a fraction $(1-p)$ is subject to the crystalline anisotropy field of intensity D. Given a configuration of the random variables, the partition function can be written as

$$Z_N = \mathrm{Tr}\exp\left[\beta\frac{J_0}{N}\sum_{\langle i,j\rangle} S_iS_j + \beta\frac{J}{N}\sum_{\langle i,j\rangle}(\xi_i\eta_j + \xi_j\eta_i)S_iS_j + \beta H\sum_i S_i + \beta\sum_i D_iS_i^2\right], \tag{2.5}$$

where $\beta = 1/k_BT$ is the inverse of temperature and $\beta J_0 = K_0$ e $\beta J = K$.

Using the following identities:

$$\sum_{\langle i,j\rangle}(\xi_i\eta_j + \xi_j\eta_i)S_iS_j = \frac{1}{2}\left[\left(\sum_i(\xi_i + \eta_i)S_i\right)^2 - \left(\sum_i \xi_iS_i\right)^2 - \left(\sum_i \eta_iS_i\right)^2 - 2\sum_i \xi_i\eta_iS_i^2\right], \tag{2.6}$$

$$\sum_{\langle i,j\rangle} S_iS_j = \frac{1}{2}\left[\left(\sum_i S_i\right)^2 - \sum_i S_i^2\right], \tag{2.7}$$

$$\exp(\lambda a^2) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}\exp\left(-\frac{x^2}{2} + ax\sqrt{2\lambda}\right)dx, \tag{2.8}$$

and the procedures employed in Refs. [24,23], the free energy functional and the equation of states for the ferromagnetic phase (magnetization) and the spin-glass phase (parameter q) are given by

$$\begin{aligned}\beta f = p\times\Bigg\{&\frac{1}{2}[K_0m^2 + 2Kq^2] - \frac{1}{4}\Bigg\{\ln\left[\sum_{m_s}\cosh[|m_s|\{K_0m + 2Kq + \beta H\}]\right]\\ &+ 2\ln\left[\sum_{m_s}\cosh[|m_s|\{K_0m + \beta H\}]\right]\\ &+ \ln\left[\sum_{m_s}\cosh[|m_s|\{K_0m - 2Kq + \beta H\}]\right]\Bigg\}\Bigg\}\\ &+ (1-p)\Bigg\{\frac{1}{2}[K_0m^2 + 2Kq^2]\\ &- \frac{1}{4}\Bigg\{\ln\left[\sum_{m_s}\exp(m_s^2\beta D)\cosh[|m_s|\{K_0m + 2Kq + \beta H\}]\right]\\ &+ 2\ln\left[\sum_{m_s}\exp(m_s^2\beta D)\cosh[|m_s|\{K_0m + \beta H\}]\right]\\ &+ \ln\left[\sum_{m_s}\exp(m_s^2\beta D)\cosh[|m_s|\{K_0m - 2Kq + \beta H\}]\right]\Bigg\}\Bigg\},\end{aligned} \tag{2.9}$$

and

$$\begin{aligned}m = \frac{1}{4}\Bigg\{&p\Bigg[\frac{\sum_{m_s}|m_s|\sinh[|m_s|\{K_0m + 2Kq + \beta H\}]}{\sum_{m_s}\cosh[|m_s|\{K_0m + 2Kq + \beta H\}]}\\ &+ 2\frac{\sum_{m_s}|m_s|\sinh[|m_s|\{K_0m + \beta H\}]}{\sum_{m_s}\cosh[|m_s|\{K_0m + \beta H\}]}\\ &+ \frac{\sum_{m_s}|m_s|\sinh[|m_s|\{K_0m - 2Kq + \beta H\}]}{\sum_{m_s}\cosh[|m_s|\{K_0m - 2Kq + \beta H\}]}\Bigg]\\ &+ (1-p)\Bigg[\frac{\sum_{m_s}\exp(m_s^2\beta D)|m_s|\sinh[|m_s|\{K_0m + 2Kq + \beta H\}]}{\sum_{m_s}\exp(m_s^2\beta D)\cosh[|m_s|\{K_0m + 2Kq + \beta H\}]}\\ &+ 2\frac{\sum_{m_s}\exp(m_s^2\beta D)|m_s|\sinh[|m_s|\{K_0m + \beta H\}]}{\sum_{m_s}\exp(m_s^2\beta D)\cosh[|m_s|\{K_0m + \beta H\}]}\\ &+ \frac{\sum_{m_s}\exp(m_s^2\beta D)|m_s|\sinh[|m_s|\{K_0m - 2Kq + \beta H\}]}{\sum_{m_s}\exp(m_s^2\beta D)\cosh[|m_s|\{K_0m - 2Kq + \beta H\}]}\Bigg]\Bigg\},\end{aligned} \tag{2.10}$$