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Effective-field theory on the kinetic spin-3/2 Ising model

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ABSTRACT

The effective-field theory (EFT) is used to study the dynamical response of the kinetic spin-3/2 Ising model in the presence of a sinusoidal oscillating magnetic field. The effective-field dynamic equations are given for the honeycomb lattices ($Z=3$). The dynamic order parameter, the dynamic quadrupole moment are calculated. We have found that the behavior of the system strongly depends on the crystal field interaction D. The dynamic phase boundaries are obtained, and there is no dynamic tricritical point on the dynamic phase transition line. The results are also compared with previous results which obtained from the mean-field theory (MFT) and the effective-field theory (EFT) for the square lattices ($Z=4$). Different dynamic phase transition lines show that the thermal fluctuations are a key factor of the dynamic phase transition.

1. Introduction

The spin-3/2 Ising model was first proposed for a qualitative description of phase transitions observed in the compound $DyVO_4$ [1]. And then the equilibrium properties of the model have been investigated by a variety of theoretical studies [2–17], such as the mean-field theory (MFT), the effective-field theory (EFT), the Monte Carlo (MC) simulations, the cluster variation method and the renormalization-group techniques.

The equilibrium properties of the spin-3/2 Ising model are well known within the framework of equilibrium statistical physics, but the dynamic phase transition properties of nonequilibrium spin-3/2 Ising model have not been well explored. Grandi et al. discussed dynamic critical exponents using the Monte Carlo simulations and short-time dynamic scaling [18]. Keskin and Canko studied the dynamic behavior of spin-3/2 Ising model through the path probability method [19,20] and the Onsager theory of irreversible thermodynamics [21,22]. Keskin et al. investigated the dynamic phase diagram of spin-3/2 Ising model under an oscillating magnetic field by the use of the mean-field theory (MFT) [23–26]. Cengiz et al. studied the ultrasonic attenuation on the Bethe lattice for a spin-3/2 Ising model using the Onsager theory of irreversible thermodynamics [27]. Vatansever et al. investigated the relaxation and complex magnetic susceptibility treatments of a spin-3/2 Ising model by a method combining the statistical equilibrium theory and the thermodynamics limit [28].

But all these works were studied in the framework of the MFT, and the transition is not truly dynamic as it can exist even in the zero frequency (static) limit of the driving field. This transition in the static limit is an artifact of the mean-field approximation, which neglects nontrivial thermal fluctuations. Efforts have been made to treat the effects of the thermal fluctuations using the EFT for a spin-3/2 Ising model under an oscillating magnetic field [29]. It shows that the dynamic tricritical behavior and multicritical points may exist in the certain region of D for the square lattices ($Z=4$). Now the problem is to find out whether there are the dynamic tricritical behavior and multicritical points in the spin-3/2 Ising model for the honeycomb lattices ($Z=3$). We know that the EFT considers partially the spin–spin correlations and results in an improvement over the MFT. Decreasing the coordination number of the Ising model, the more thermal fluctuations are taken into account in the EFT, and the more accurate results will be obtained.

Therefore in this work, we use the correlated effective-field theory, as an analytical method, to study the kinetic spin-3/2 Ising model on a honeycomb lattice and compare the results with those of recently published works. The layout of this paper is as follows. In Section 2, we briefly present the EFT method we used. The results and discussion are presented in Section 3. In Section 4, we summarize our conclusions.

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2. Formulation

We consider a kinetic spin-3/2 Ising model with N sites described by the Hamiltonian given by

$$\mathcal{H} = -\sum_{<i,j>} J_{ij} s_i s_j - D\sum_i s_i^2 - h(t)\sum_i s_i \quad (1)$$

Here $s_i = \pm\frac{1}{2}, \pm\frac{3}{2}$ represents the spin variable at site i, J_{ij} represents the spin–spin interaction strength between sites i and j, the sum Σ_i is carried out over all the sites, the sum $\Sigma_{<i,j>}$ is carried out over all the distinct nearest-neighbor pairs, D is the crystal field interaction, and $h(t)$ is a time-dependent external field given by $h(t) = h_0 \sin(\omega t)$. The system is in contact with an isothermal heat bath at temperature T. For simplicity all J_{ij} are taken equal to a constant $J > 0$.

The system evolves according to the Glauber stochastic process [30] at a rate of $1/\tau$ transitions per unit time. From the master equation associated to the stochastic process, it follows that the average magnetization satisfies the following equation

$$\tau\frac{d\langle s_i\rangle}{dt} = -\left(\langle s_i\rangle - \left\langle\frac{3\sinh(3\beta a/2) + \exp(-2\beta D)\sinh(\beta a/2)}{2\cosh(-3\beta a/2) + 2\exp(-2\beta D)\cosh(-\beta a/2)}\right\rangle\right), \quad (2)$$

where $\beta = 1/k_B T$ and $a = J\sum_j s_j + h(t)$, k_B is the Boltzmann constant and $\tau = 1$.

According to the effective-field theory with correlations, as was initiated by Honmura and Kaneyoshi [31], it is convenient to introduce the differential operator technique into the expression. And then for spin-3/2 we introduce the generalized but approximated Van der Waerden identity [32], we obtain

$$\frac{dm}{dt} = -m + [\cos h(J\eta\nabla) + \frac{m}{\eta}\sin h(J\eta\nabla)]^Z f(x+h)|_{x=0}, \quad (3)$$

$$\frac{dq}{dt} = -q + [\cos h(J\eta\nabla) + \frac{m}{\eta}\cos h(J\eta\nabla)]^Z g(x+h)|_{x=0}, \quad (4)$$

where $\nabla = \partial/\partial x$ is a differential operator, $m = \langle s_i\rangle$ represents the average magnetization, $q = \eta^2 = \langle s_i^2\rangle$ represents the quadrupole moment. Z is the coordination number. The two functions $f(x+h)$ and $g(x+h)$ are expressed as

$$f(x+h) = \frac{3\sinh[3\beta(x+h)/2] + \exp(-2\beta D)\sinh[\beta(x+h)/2]}{2\cosh[3\beta(x+h)/2] + 2\exp(-2\beta D)\cosh[\beta(x+h)/2]}, \quad (5)$$

$$g(x+h) = \frac{9\cosh[3\beta(x+h)/2] + \exp(-2\beta D)\cosh[\beta(x+h)/2]}{4\cosh[3\beta(x+h)/2] + 4\exp(-2\beta D)\cosh[\beta(x+h)/2]}, \quad (6)$$

For the honeycomb lattices $Z = 3$, the set of the dynamic equations are given as

$$\frac{dm}{dt} = -m + A_0(\eta) + A_1(\eta)m + A_2(\eta)m^2 + A_3(\eta)m^3, \quad (7a)$$

$$\frac{dq}{dt} = -q + B_0(\eta) + B_1(\eta)m + B_2(\eta)m^2 + B_3(\eta)m^3. \quad (7b)$$

The coefficients $A_i\,(i = 0-3)$ and $B_i\,(i = 0-3)$ can be easily calculated employing a mathematical relation $\exp(a\nabla)f(x) = f(x+a)$. And the temperature T, the longitudinal field h and the crystal field interaction D are measured in units of ZJ. The frequency of the longitudinal field is $\omega = 2\pi$. Eqs. (7) can be solved using the fourth-order Runge–Kutta method.

The dynamic order parameter M and the dynamic quadrupole moment Q are defined as

$$M = \frac{\omega}{2\pi}\oint m(t)dt \text{ and } Q = \frac{\omega}{2\pi}\oint q(t)dt. \quad (8)$$

the two types of solutions can be identified: a symmetric one where $m(t)$ follows the field oscillating around zero giving $M = 0$, and an antisymmetric one where $m(t)$ does not follow the field and oscillates around a finite value different from zero, such that $M \neq 0$; if it oscillates around $\pm(3/2)$, this antisymmetric solution corresponds to the ferromagnetic–(3/2) phase ($F_{3/2}$ phase); if it oscillates around $\pm(1/2)$, this antisymmetric solution corresponds to the ferromagnetic–(1/2) phase ($F_{1/2}$ phase).

3. Results and discussion

By solving the effective-field Eqs. (7), the $m(t)-t$ and $q(t)-t$ curves are obtained. Then the dynamic order parameter M and the dynamic quadrupole moment Q can be calculated using the above definitions. The temperature variations of M and Q are shown in Fig. 1. The dynamic transition point is identified as the temperature at which the dynamic order parameter M vanishes. If the dynamic order parameter M decreases to zero continuously as the temperature increases, the system undergoes the second-order phase transition; if the dynamic order parameter M jumps to zero discontinuously as the temperature increases, the system undergoes the first-order phase transition. Fig. 1(a) shows that the system exhibits a second-order phase transition from the $F_{3/2}$ phase to the P phase for $D = -0.25$ and $h_0 = 0.5$. The dynamic phase transition temperature $T_c/ZJ = 0.705$ can be compared with the MFT result $T_c/ZJ = 0.915$ in Ref. [23]. Fig. 1(b) and (c) illustrates the thermal variations of M and Q for $D = -0.475$ and $h_0 = 0.125$ for two different initial values. From Fig. 1(b) we can see that the system shows the first-order phase transition from the $F_{1/2}$ phase to the $F_{3/2}$ phase for the dynamic phase transition temperature $T_t/ZJ = 0.15$ and then shows the second-order phase transition from the $F_{3/2}$ phase to the P phase for the dynamic phase transition temperature $T_c/ZJ = 0.5205$. The results can be compared with the MFT results $T_t/ZJ = 0.2$ and $T_c/ZJ = 0.5575$ in Ref. [23]. Fig. 1 (d) and (e) illustrates the thermal variations of M and Q for $D = -0.5$ and $h_0 = 0.25$ for two different initial values. In Fig. 1 (d) the system shows the first-order phase transition from the $F_{3/2}$ phase to the $F_{1/2}$ phase at $T_t/ZJ = 0.21$ and then shows the second-order transition from the $F_{1/2}$ phase to the P phase at $T_c/ZJ = 0.305$. The results can be compared with the MFT results $T_t/ZJ = 0.2$ and $T_c/ZJ = 0.38$ in Ref. [23]. Fig. 1(f) and (g) illustrates the thermal variations of M and Q for $D = -0.625$ and $h_0 = 0.1$ for two different initial values. From Fig. 1(f) the system shows the first-order phase transition from the $F_{3/2}$ phase to the $F_{1/2}$ phase at $T_t/ZJ = 0.057$ and then shows the second-order phase transition from the $F_{1/2}$ phase to the P phase at $T_c/ZJ = 0.179$. In contrast, the system exhibited three successive phase transitions for $D = -0.625$ and $h_0 = 0.1$ using the EFT for the square lattices of Ref. [29], and the dynamic phase transition temperatures were $T_t/ZJ = 0.095$, $T_{c1}/ZJ = 1.095$, and $T_{c2}/ZJ = 1.125$, respectively. Fig. 1(h) shows that the system undergoes a second-order phase transition from the $F_{1/2}$ phase to the P phase for $D = -0.85$ and $h_0 = 0.1$. The dynamic phase transition temperature is $T_c/ZJ = 0.173$.

Comparing with previous results obtained by the MFT of Ref. [23] and the EFT of Ref. [29], the critical temperatures of the MFT are higher than the ones given by the EFT except for the first-order transition point at $D = -0.5$ and $h_0 = 0.25$. Partially thermal fluctuations are considered in the EFT but lacking all of thermal fluctuations in the MFT, and these results given above indicate that the thermal fluctuations play an essential role in the dynamic phase transition. Some of interesting behaviors found by the mean-field approximation may disappear if the thermal