Journal of Magnetism and Magnetic Materials 393 (2015) 284–292

Contents lists available at ScienceDirect

Journal of Magnetism and Magnetic Materials

journal homepage: www.elsevier.com/locate/jmmm

Simultaneous effects of MHD and partial slip on peristaltic flow of Jeffery fluid in a rectangular duct

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ARTICLE INFO

Article history:
Received 24 April 2015
Received in revised form
12 May 2015
Accepted 24 May 2015
Available online 29 May 2015

Keywords:
Magnetohydrodynamics
Peristaltic flow
Jeffery fluid
Partial slip
Exact solutions

ABSTRACT

The purpose of this paper is to study the closed-form solutions of peristaltic flow of Jeffery fluid under the simultaneous effects of magnetohydrodynamics (MHD) and partial slip conditions in a rectangular duct. The influence of wave train propagation is also taken into account. The analysis of mathematical model consists of continuity and the momentum equations are carried out under long wavelength $(0 < < \to \infty)$ and low Reynolds number $(\mathrm{Re} \to 0)$ assumptions. The governing equations are first reduced to the dimensionless system of partial differential equation using the appropriate variables and afterwards exact solutions are obtained by applying the method of separation of variables. The role of pertinent parameters such as Hartmann number M, slip parameter β_1, volumetric flow rate Q, Jeffery parameter λ_1 and the aspect ratio β against the velocity profile, pressure gradient and pressure rise is illustrated graphically. The streamlines have also been presented to discuss the trapping bolus discipline. Comparison with the existing studies is made as a limiting case of the considered problem.at the end.

1. Introduction

Over the last thirty years, or so, a very hot topic, namely non-Newtonian fluid flow appeared in many application in industrial processes, biology, medicine, catalytic chemistry and environmental applications [1–6]. In bio-fluids the physiological system has been investigated by several researchers in order to find the treatment of diagnostic problems that arise during circulation in a human body. There are several models which have been proposed to describe such physiological fluids, however their full potential has not been exploited yet and a lot of questions remain unresolved. Among several models, non-Newtonian Jeffrey model is significant because Newtonian fluid model can be deduced from it as a special case by taking $\lambda_1 = 0$. It is also speculated that the physiological fluids such as blood exhibit Newtonian and non-Newtonian behaviors simultaneously.

The peristaltic models with reactions are also encountered in the physiological study that is why among all continuous system taking place in a human's body and becoming the main reason for one's life, peristalsis is one of them. Basically peristaltic word comes from Greek word peristaltikos which means clasping and compressing. The importance of peristalsis can be assumed by the fact that, it is an automatic series of muscular contraction and relaxation, which takes place in human's body, such as digestive tract or digestive system. Peristalsis does cause the movement of food through digestive system, chyme in the gastrointestinal tract, urine from kidney to bladder and bile from gallbladder to duodenum. Also, transport of lymph in the lymphatic vessels and vasomotion of small blood vessels like arterioles, venules and capillaries involve the peristaltic motion are the common examples of peristaltic motion. In addition peristaltic motion has been equally, playing an important and beneficial part in physiological sciences by devising mechanical and biomechanical instruments such as roller pumps and heart lung machine, which function mainly by using the principle of peristalsis. This has opened a new dimension for researchers to maneuver their equipment for obtaining better results in their respective field of interest. Different mathematical models have been employed by many authors to use peristaltic flow in Newtonian and non-Newtonian fluid [7–11].

Moreover, magnetohydrodynamics (MHD) effects on peristalsis are significant in magneto therapy, hyperthermia, arterial flow, compressor, etc. The controlled application of low intensity and frequency pulsating fields modify the cell and tissue. Magnetic susceptibility of chyme is also satisfied the ions contained in the chyme or with heat generated by the magnetic field. The magnets could heat inflammations, ulceration, several diseases of uterus and bowel (intestine). Furthermore, MHD is used in the study of

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http://dx.doi.org/10.1016/j.jmmm.2015.05.071
0304-8853/

electrically conducting fluids; examples of such fluids include electrolytes, saltwater, liquid metals and plasmas. A number of researchers have applied magnetic field on peristaltic mechanisms, one way or the other. An extensive literature on the said topics is now available but we can only mention a few recent interesting investigations here [12–19].

Furthermore, the fluids that exhibit slip effect have many applications, for instance, the polishing of artificial heart valves and internal cavities. In all these studies the peristaltic flow problems have been extensively investigated with no slip condition. A very less emphasis has been given to such flows in the presence of a slip/partial slip conditions. Although, the application of slip condition in the peristaltic flows has special relevance in polymers and physiology. The case of slip effects may appear for two types of fluids namely fluids having much more elastic character and rare field gases. In these fluids, slippage appears subject to large tangential traction. It is found through experimental observations that the existence of slippage is possible in the non-Newtonian fluids, polymer solution and molten polymer. Additionally, a clear layer is occasionally detected next to the wall when flow of dilute suspension of particles examined. In experimental physiology such a layer is observed when blood flow through capillary vessels is considered. The peristaltic flows subject to different flow aspects and configurations have been examined. Some pertinent studies in numerous situations can be seen from the list of Refs. [20–31] and several therein.

With all abovementioned studies, one can clearly observe that no analysis for magnetohydrodynamic peristaltic flow of Jeffery fluid in the presence of partial slip conditions is reported in the available literature yet. The present work puts forward to fill this gap. The flow analysis is performed under the constraints of long wavelength approximation and low Reynolds number. An exact solution for the expression for both velocity of the fluid and pressure gradient are obtained by using the method of separation of variables. The pumping characteristics such as pressure rise and trapping phenomena are obtained numerically by using software Mathematica. The physical features of pertinent parameters have been comprehensively elaborated through graphs. The organization of paper is in the following fashion: After introduction, Section 2 contains the formulation of the problem. Solution of the problem is given in Section 3. Section 4 is devoted for results and discussion and finally the main outcomes are concluded in Section 5.

2. Mathematical formulation

Let us consider the peristaltic flow of an incompressible, Jeffrey fluid in a duct of rectangular cross section having the channel width $2d$ and height $2a$. Cartesian coordinates system is considered in such a way that $X-$axis is taken along the axial direction, $Y-$axis is taken along the lateral direction and $Z-$axis is along the vertical direction of a rectangular duct.

The peristaltic waves on the wall are represented as

$$Z = H(X,\ t) = \pm a \pm b\ \cos\left[\frac{2\pi}{\lambda}(X - ct)\right], \tag{1}$$

where a and b are the amplitudes of the waves, λ is the wave length, c is the velocity of propagation, t is the time and X is the direction of the wave propagation. The walls parallel to $XZ-$plane remain undisturbed and are not subject to any peristaltic wave motion. As there is no change in lateral direction of the duct cross section therefore, it is assumed that the lateral velocity is zero. The basic equations governing the flow of an incompressible are given by

$$\nabla.\ V = 0, \tag{2}$$

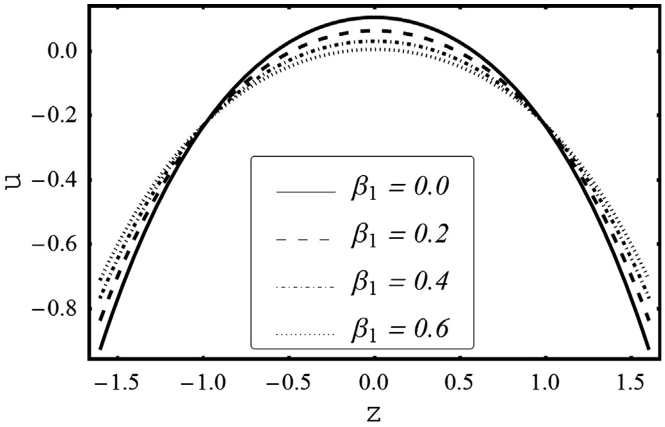

Fig. 1. Velocity profile for different values of β_1.

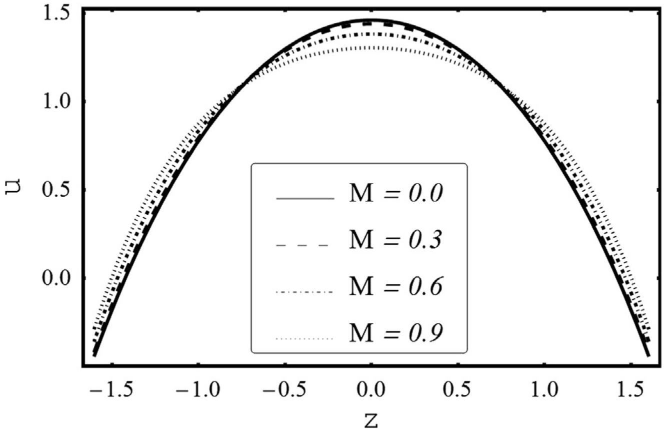

Fig. 2. Velocity profile for different values of M.

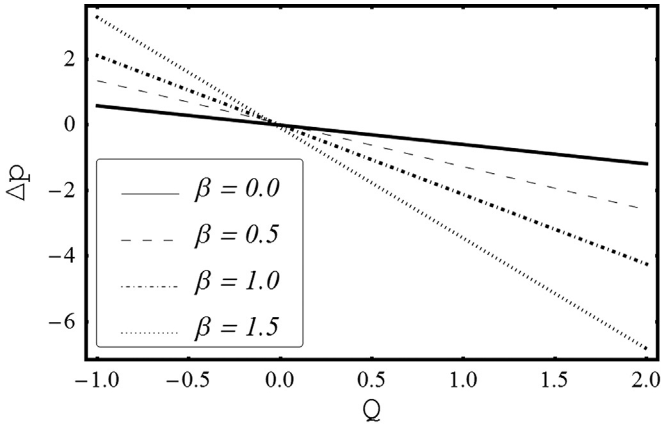

Fig. 3. Variation of Δp with Q for different values of β.

$$\rho\frac{DV}{Dt} = \nabla.\ T + J\times B, \tag{3}$$

In which ρ is the density of the fluid, V is the velocity of the fluid, D/Dt is the material derivative, B is the magnetic field and T is the Cauchy stress tensor for Jeffrey fluid which is defined as

$$T = -pI + S \tag{4}$$

where extra stress tensor S is given by