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Stability of skyrmions on curved surfaces in the presence of a magnetic field

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ABSTRACT

We study the stability and energetics associated to skyrmions appearing as excitations on curved surfaces. Using a continuum model we show that the presence of cylindrically radial and azimuthal fields destabilize the skyrmions that appear in the absence of an external field. Weak fields generate fractional skyrmions while strong magnetic fields yield stable 2π-skyrmions, which have their widths diminished by the magnetic field strength. Under azimuthal fields vortex appears as stable state on the curved surface.

1. Introduction

During the last decade a strong interest has focused on the properties of topological structures on curved surfaces based on the fact that the surface's shape determines the physical properties of several systems. As examples, one can cite the ordering of nematic liquid crystals on a curved substrate [1] and the local density of states of a conical graphene sheet [2]. In the same hand, curvature is important to describe the physical properties of magnetic nanostructures. In a recent work by Carvalho-Santos et al., it was shown that toroidal nanorings can support a vortex-like magnetization for smaller sizes than cylindrical nanorings [3].

Regarding experiments, curved nanostructures have been recently prepared. As examples we can mention the production of permalloy caps on non-magnetic spheres [4] and permalloy cylindrically curved magnetic segments with different radii of curvature on non-magnetic rolled-up membranes [5]. Moreover, the synthesis of periodically modulated nanowires, which can be controlled by external parameters that allow control of the geometry of pores has been reported in several works (see [6] and references therein).

Skyrmions were first described in nuclear physics, when Skyrme showed that topologically stable field configurations for interacting pions can occur as particle-like solutions [7]. However, these topological structures do not appear only in particle physics. In the last two decades, skyrmions have been studied in several condensed matter systems, e.g., nematic liquid crystals [8], Bose–Einstein condensates [9], and magnetic systems (see Refs. [10,11] and references therein). In particular, regarding magnetic systems, skyrmions are chiral spin structures with a whirling configuration which appear like a magnetization groundstate due to the competition among Exchange, anisotropy, Zeeman and Dzyaloshinskii–Moriya interactions [11]. Otherwise, skyrmions may appear as excited state of the continuum Heisenberg Model (HM), which consists of the non-linear σ-model if the constraint $m^2 = 1$ is considered for the spin space [12]. These spin collective modes have topological stability since its structure cannot be continuously deformed to a ferromagnetic or other magnetic state. When present in curved systems, the energy, stability and width associated to skyrmion-like excitations depend on the surface's curvature [13–18].

Related to curved systems, it has been previously shown that in the presence of an axial magnetic field 2π-skyrmions appear in simply and non-simply connected surfaces [19–21]. Furthermore, a fractional skyrmion-like pattern can be induced in curved nanomagnets with a vortex state when an external magnetic field acts into the system [22]. Based on these ideas, in this paper we extend these results by studying the excitations appearing on magnetic surface with rotational symmetry under a radial or an azimuthal magnetic field. Our results show that a weak magnetic field can breakout the skyrmion stability by shifting the energy minima in the absence of fields. On the other hand, a strong magnetic field coupled to the curvature of the surface can lead to the appearance

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of 2π-skyrmions. The magnitude and direction of the magnetic field play an important role on properties such as the phase, the width and the energy of the skyrmions.

This work is divided as follows: in Section 2 we present a discussion on the Heisenberg Model on cylindrically symmetric surfaces in the absence and presence of an external magnetic field. In Section 3, we present the derived equations coming from the interaction with cylindrically radial (RMF) and azimuthal magnetic fields (AMF). Section 4 presents an analytical study of the appearance and stability of 2π-skyrmions due to the interaction with AMF and RMF. Finally, in Section 5 we present the conclusions and prospects of this work.

2. Continuum Heisenberg Model on cylindrically symmetric surfaces

A spin system lying on a surface embedded in a 3D-space can be described by a Heisenberg Model in a continuum approximation of spatial and spin variables, valid at sufficiently large wavelength and low temperature. Then, and assuming only exchange terms, we can describe our spin system by the following Hamiltonian [13]:

$$H = J \iint g^{\mu\nu} \partial_\mu m^\alpha \partial_\nu m_\alpha \; dA, \tag{1}$$

where $\partial_{\mu,\nu} \equiv \partial/\partial\eta^{\mu,\nu}$. The surface is described by the curvilinear coordinates η_1 and η_2, $dA = \sqrt{|\det[g_{\mu\nu}]|}\, d\eta_1\, d\eta_2$ is the surface element, $g^{\mu\nu}$ is the surface contravariant metric. Repeated indices must be summed, with μ and ν varying from 1 to 2 and α varying from 1 to 3. J denotes the coupling between neighboring spins, and according to $J < 0$ or $J > 0$, the Hamiltonian describes a ferro or antiferromagnetic system. The classical spin vector field is given by $\vec{m} = (\sin\Theta\cos\Phi, \sin\Theta\sin\Phi, \cos\Theta)$, so that $\Theta = \Theta(\eta_1, \eta_2)$ and $\Phi = \Phi(\eta_1, \eta_2)$.

Our main interest is to study static topological excitations appearing when we consider a rotationally symmetric surface in the presence of external fields. In cylindrical coordinate system, an arbitrary surface with rotational symmetry can be parametrized by $\mathbf{r} = (\rho, \phi, z(\rho))$, where ρ is the radius of the surface at height z, and ϕ accounts for the azimuthal angle (see Fig. 1). Thus, the Hamiltonian (1) can be rewritten as

$$H = J \iint \left\{ \sqrt{\frac{g_{\phi\phi}}{g_{\rho\rho}}} \left[(\partial_\rho \Theta)^2 + \sin^2\Theta (\partial_\rho \Phi)^2 \right] + \sqrt{\frac{g_{\rho\rho}}{g_{\phi\phi}}} \left[(\partial_\phi \Theta)^2 + \sin^2\Theta (\partial_\phi \Phi)^2 \right] \right\} d\rho\, d\phi. \tag{2}$$

The Euler–Lagrange equations (ELE) derived from (2) are

$$2(\partial_\zeta^2 \Theta + \partial_\phi^2 \Theta) = \sin 2\Theta \left[\left(\partial_\zeta \Phi\right)^2 + \left(\partial_\phi \Phi\right)^2 \right] \tag{3}$$

and

$$\sin^2\Theta \left(\partial_\zeta^2 \Phi + \partial_\phi^2 \Phi \right) + \sin 2\Theta \left(\partial_\zeta \Theta \partial_\zeta \Phi + \partial_\phi \Theta \partial_\phi \Phi \right) = 0. \tag{4}$$

In these expressions $d\zeta = \sqrt{g_{\rho\rho}/g_{\phi\phi}}\, d\rho$ is a length scale depending on the geometric parameters of the underlying manifold and we have used the fact that $\partial_\phi(\sqrt{g_{\phi\phi}/g_{\rho\rho}}) = 0$, since $g_{\phi\phi} = \rho^2$ and $g_{\rho\rho} = 1 + (dz/d\rho)^2$ have no dependence on ϕ.

The set of equations describing the spin vector field presents a length scale parameter that depends on the considered geometry, leading to shape induced changes in the energy and the stability of the excitations of our spin system. When a spin vector field with cylindrical symmetry is considered, that is, $\Phi(\rho, \phi) \equiv \Phi(\phi)$ and $\Theta(\rho, \phi) \equiv \Theta(\rho)$, the above set of equations is simplified and the sine-Gordon system is obtained. Its solution is a topological π-skyrmion, represented by the spin profile given by $\Phi = \phi + \phi_0$ and

$$\Theta_{\rm iso} = 2\arctan(e^{\zeta - \zeta_0}), \tag{5}$$

where ϕ_0 and ζ_0 are phases depending on the boundary conditions and that does not account to the energy calculations. If a skyrmion profile is given by a function $f(\lambda\zeta)$, its width is given by λ^{-1}, which generally appears in front of the $\sin 2\Theta$ term of Eq. (3). However, due to the chosen parametrization of the surface, the skyrmion width is rescaled to unity. Nevertheless, the curvature dependence of the skyrmion width can be evidenced by calculating $\zeta = \int \sqrt{g_{\rho\rho}/g_{\phi\phi}}\, d\rho$.

Once the minima associated to Eq. (2) are given by $\Theta = n\pi$, with n being an integer, the π-skyrmion appears as a continuous transition connecting the two neighboring minima 0 and π. This transition consists of a topological excitation, in which the spin sphere is obtained when the underlying manifold is mapped, so belonging to the first class of the second homotopy group. Despite the ferromagnetic state is less energetic than the skyrmionic one, there is a very high energy barrier separating these states and the skyrmion cannot be deformed in a ferromagnetic-like state by a continuous variation of the order parameter, characterizing it as a

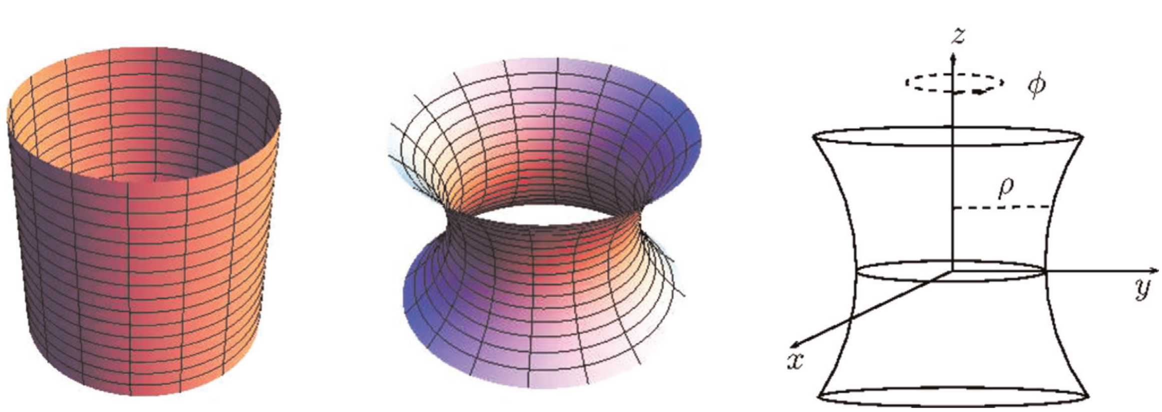

Fig. 1. Examples of surface with cylindrical symmetry, a cylinder and a catenoid. At right, we show the representation of the cylindrical-like coordinate system of a surface with rotational symmetry. ϕ is the azimuthal angle and ρ is the radius of the surface at the height z.