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Thermally radiative three-dimensional flow of Jeffrey nanofluid with internal heat generation and magnetic field

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ABSTRACT

This research work addresses the three-dimensional hydromagnetic flow of Jeffrey fluid with nanoparticles. Flow is generated by a bidirectional stretching surface. The effects of thermal radiation and internal heat generation are encountered in energy expressions. More realistic convective boundary conditions at the surface are employed instead of constant surface temperature and mass species conditions. Boundary layer assumptions lead to the governing non-linear mathematical model. Resulting equations through momentum, energy and mass species are made dimensionless using suitable variables. The solution expressions of dimensionless velocities, temperature and nanoparticle concentration have been computed for the convergent series solutions. The impacts of interesting parameters on the dimensionless quantities are displayed and interpreted. The values of physical quantities are computed and analyzed.

1. Introduction

It is well known that the industrial fluids commonly differ from the viscous fluid in view of their diverse rheological characteristics. Such type of fluids falls into the category of non-Newtonian liquids which are quite interesting in different applications. The examples of such applications include plastics manufacturing, wire and blade coating, dying of paper and textile, polymer industries, food processing, movement of biological fluids and many others. The simple Navier–Stokes equations are not suitable to characterize the flow behavior of non-Newtonian liquids. There does not exist a single relation which can predict the characteristics of all the non-Newtonian materials. Thus the different types of non-Newtonian models are suggested in the literature. The present fluid model is known as the Jeffrey fluid. This model is the linear viscoelastic fluid and has advantage on Maxwell model because of its extra feature about ratio of relaxation to retardation times. Turkyilmazoglu and Pop [1] reported the two-dimensional stagnation point flow of Jeffrey fluid. They developed both exact and analytical solutions for the considered flow equations. Hayat et al. [2] discussed the impact of thermal and solutal stratification in mixed convection flow of Jeffrey fluid. Entropy analysis for two-dimensional hydromagnetic flow of Jeffrey fluid with nanoparticles is presented by Dalir et al. [3]. The authors here developed the Keller-Box algorithm for the solution of governing nonlinear ordinary differential system. Hayat et al. [4] investigated the magnetohydrodynamic flow of Jeffrey fluid past a convectively heated plate. Three-dimensional boundary layer flow of Jeffrey fluid over a bidirectional stretching surface with Robin's condition is addressed by Shehzad et al. [5].

The particular physical characteristics of nanofluids make their potential role in broad range of industrial applications like ceramics, drug delivery, paints and coatings, food products etc. [6,7]. The researchers and engineers at present paid special attention to nanofluids due to their enhancement in heat transfer characteristics. Such enhancement in heat transfer opens new doors of applications of nanofluids which include car engines, super coolant in nuclear reactors, radiators, X-rays, computers and many others. The absorption of heat rate of nanofluid is higher than any ordinary base liquid due to which it is known as super-coolant that decreases the size of system and enhances its performance. Nanofluid increases the thermal conductivity of ordinary liquids dramatically which is always stable and no additional deficiency like erosion, pressure drop and redimentation [8]. The applied magnetic field effects are commonly arisen in physics and

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engineering problems. Many industrial equipment including MHD generators, pumps, bearings and boundary layer control are seriously affected due to the interaction of electrically conducting liquid and magnetic field. Further the flows of electrically conducting nanofluids have diverse usages in wound treatment, gastric medications, sterilized devices, elimination of tumors, asthma treatment, magnetic resonance imaging etc. The important studies on nanofluids with magnetic field and without magnetic field can be seen in the refs. [9–20].

Here our theme is to address the magnetohydrodynamic three-dimensional flow of Jeffrey fluid in presence of Brownian motion and thermophoresis effects. Thermally radiative flow in presence of internal heat generation is taken into account. Convective heat and mass condition are imposed at the boundaries of the surface. All the above cited work dealt with constant surface temperature and concentration or thermal convective condition but here we consider both temperature and mass convective conditions. Such conditions are more adequate and realistic to discuss the physical phenomenon. Boundary layer approximations lead to the nonlinear partial differential system which reduces to the ordinary differential equations via transformation technique. Homotopy analysis method [21–30] has been employed to find solution of dimensionless quantities. Physical interpretation and convergence of the obtained solutions are discussed.

2. Governing problems

We consider the three-dimensional laminar flow of Jeffrey nanofluid induced by a stretched surface. The ambient fluid temperature and nanoparticle concentration are considered to be T_∞ and C_∞. The thermal radiation and internal heat generation effects are retained in the energy equation. Induced magnetic field is not taken into account due to smaller magnetic Reynolds number. Electric field is assumed to be zero. The present boundary layer equation for three-dimensional flow of Jeffrey nanofluid can be expressed as follows:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0, \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+w\frac{\partial u}{\partial z}=\frac{\nu}{1+\lambda_1}\left(\frac{\partial^2 u}{\partial z^2}+\lambda_2\left(\begin{array}{c}\frac{\partial u}{\partial z}\frac{\partial^2 u}{\partial x\partial z}+\frac{\partial v}{\partial z}\frac{\partial^2 u}{\partial y\partial z}+\frac{\partial w}{\partial z}\frac{\partial^2 u}{\partial z^2}\\ +u\frac{\partial^3 u}{\partial x\partial z^2}+v\frac{\partial^3 u}{\partial y\partial z^2}+w\frac{\partial^3 u}{\partial z^3}\end{array}\right)\right)-\frac{\sigma B_0^2}{\rho_f}u, \tag{2}$$

$$u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}+w\frac{\partial v}{\partial z}=\frac{\nu}{1+\lambda_1}\left(\frac{\partial^2 v}{\partial z^2}+\lambda_2\left(\begin{array}{c}\frac{\partial u}{\partial z}\frac{\partial^2 v}{\partial x\partial z}+\frac{\partial v}{\partial z}\frac{\partial^2 v}{\partial y\partial z}+\frac{\partial w}{\partial z}\frac{\partial^2 v}{\partial z^2}\\ +u\frac{\partial^3 v}{\partial x\partial z^2}+v\frac{\partial^3 v}{\partial y\partial z^2}+w\frac{\partial^3 v}{\partial z^3}\end{array}\right)\right)-\frac{\sigma B_0^2}{\rho_f}v, \tag{3}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}+w\frac{\partial T}{\partial z}=\alpha\frac{\partial^2 T}{\partial z^2}+\tau(D_B\frac{\partial C}{\partial z}\frac{\partial T}{\partial z}+\frac{D_T}{T_\infty}(\frac{\partial T}{\partial z})^2)$$
$$+\frac{1}{(\rho c)_f}\frac{16\sigma^* T_\infty^3}{3k^*}\frac{\partial^2 T}{\partial z^2}+Q(T-T_\infty), \tag{4}$$

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}+w\frac{\partial C}{\partial z}=D_B\frac{\partial^2 C}{\partial z^2}+\frac{D_T}{T_\infty}\frac{\partial^2 T}{\partial z^2}, \tag{5}$$

where the respective velocity components in the $x-$, $y-$, and $z-$directions are denoted by u, v and w, λ_1, the ratio of relaxation to retardation times, λ_2 the retardation time, T the fluid temperature, $\nu = (\mu/\rho)$ the kinematic viscosity, μ the dynamic viscosity of fluid, ρ_f the density of fluid, σ the electrical conductivity, α the thermal diffusivity, Q the heat generation parameter, $\tau = \frac{(\rho c)_p}{(\rho c)_f}$ the ratio of nanoparticle heat capacity and the base fluid heat capacity, σ^* the Stefan–Boltzmann constant, k^* the mean absorption coefficient, D_B the Brownian diffusion coefficient and D_T the thermophoretic diffusion coefficient.

The boundary conditions can be written in the following forms:

$$u=ax,\ v=by,\ w=0,\ -k\frac{\partial T}{\partial z}=h_1(T_f-T),\ -D_B\frac{\partial C}{\partial z}$$
$$=h_2(C_f-C)\text{at}z=0, \tag{6}$$

$$u\to 0,\ v\to 0,\ T\to T_\infty,\ C\to C_\infty\text{as}z\to\infty, \tag{7}$$

where k denotes the thermal conductivity of fluid, a and b the constants and have dimension inverse of time and h_1 and h_2 are the heat and mass transfer coefficients.

By employing the following variables

$$u=axf'(\eta),\ v=ayg'(\eta),\ w=-\sqrt{a\nu}(f(\eta)+g(\eta)),$$
$$\eta=z\sqrt{\frac{a}{\nu}},\ \theta(\eta)=\frac{T-T_\infty}{T_f-T_\infty},\ \varphi(\eta)=\frac{C-C_\infty}{C_f-C_\infty}, \tag{8}$$

Eq. (1) is satisfied automatically and Eqs. (2)–(7) can be converted in the following forms:

$$f'''+(1+\lambda_1)((f+g)f''-f'^2)+\beta_1(f''^2-(f+g)f''''-g'f''')$$
$$-(1+\lambda_1)Mf'=0, \tag{9}$$

$$g'''+(1+\lambda_1)((f+g)g''-g'^2)+\beta_1(g''^2-(f+g)g''''-f'g''')$$
$$-(1+\lambda_1)Mg'=0, \tag{10}$$

$$(1+\frac{4}{3}Rd)\,\theta''+\Pr(f+g)\theta'+N_b\theta'\varphi'+N_t\theta'^2+\Pr S\theta=0, \tag{11}$$