Journal of Magnetism and Magnetic Materials 397 (2016) 233–239

Contents lists available at ScienceDirect

Journal of Magnetism and Magnetic Materials

journal homepage: www.elsevier.com/locate/jmmm

Mechanically induced magnetic diffusion in cylindrical magnetoelastic materials

Justin J. Scheidler, Marcelo J. Dapino *

Department of Mechanical and Aerospace Engineering, The Ohio State University, 201 W. 19th Ave., Columbus, OH 43210, USA

ARTICLE INFO

Article history:
Received 29 April 2015
Received in revised form
5 August 2015
Accepted 19 August 2015
Available online 22 August 2015

Keywords:
Magnetic diffusion
Eddy currents
Magnetostrictive materials
Ferromagnetic shape memory alloys
Dynamic stress
Dynamic strain

ABSTRACT

This paper considers the radial dependence of magnetic diffusion in cylindrical magnetoelastic materials that results from the simultaneous application of a constant surface magnetic field and a dynamic mechanical input. Mechanically induced magnetic diffusion is particularly pronounced in materials that exhibit a strong magnetoelastic coupling, such as magnetostrictive materials and ferromagnetic shape memory alloys. Analytical time- and frequency-domain solutions of the PDE governing the radial diffusion of magnetic field are derived. The solutions are non-dimensionalized by deriving a skin depth and cut-off frequency for mechanically induced diffusion, which are about 2.08 and 4.34 times those for field-induced diffusion, respectively. It is shown that the effects of mechanically induced diffusion can be incorporated in linear constitutive models through the use of a complex-valued, frequency-dependent magnetoelastic coupling coefficient and Young's modulus. The solutions show that for forcing frequencies f up to about the cut-off frequency, the magnitude of the steady-state, dynamic field increases in proportion to f. As forcing frequency increases above that range, the magnitude overshoots its high frequency limit, peaks, then decreases to its high frequency limit, at which point the dynamic magnetic flux becomes zero and continued increases in forcing frequency have no effect. Together, the derived frequency responses, skin depth, and cut-off frequency can be used to design magnetoelastic systems and determine if lamination of the magnetoelastic material is necessary

1. Introduction

Eddy currents inside electrically conducting media alter the propagation of magnetic fields into the media; the resulting attenuation and phase lag of the magnetic fields is quantified by magnetic diffusion laws. Magnetic diffusion in ferromagnets caused by the application of dynamic magnetic fields is a classical problem that has received significant attention since the late 1800s [1–3]. The influence of magnetoelasticity and static stress on field-induced magnetic diffusion has been investigated only more recently [4–6].

Dynamic mechanical inputs cause a diffusion of static magnetic fields into electrically conducting magnetoelastic materials, particularly ones that exhibit strong coupling, such as magnetostrictive materials and ferromagnetic shape memory alloys. Mechanically induced magnetic diffusion is critically important for applications in which these materials operate under dynamic mechanical loading, including dynamic sensors, energy harvesters, vibration dampers, and stiffness tuning devices. However, only a few studies on this effect have been reported.

The effects of 1D mechanically induced magnetic diffusion have been briefly studied numerically [7–9]. Sarawate and Dapino [7] investigated the magnetic field in a Ni–Mn–Ga rod and illustrated the dependence of the field's time-domain response on the radial coordinate and strain frequency for a small range of parameters. 1D mechanically induced magnetic diffusion has been analytically treated in the context of magnetostrictive energy harvesters by Davino et al. [10], who derived an expression for average harvested power, and by Zhao and Lord [11], who derived an expression for the effective internal magnetic field. However, the spatial and frequency dependence of the internal magnetic field or magnetic flux density have not been derived. Further, calculation of a skin depth and cut-off frequency for this effect are absent from the literature.

This paper presents an analytical model of linear, 1D mechanically induced magnetic diffusion in cylindrical magnetoelastic materials. The model is used to quantify the radial dependence of internal magnetic fields created by eddy currents that result from the application of harmonic, axial stresses. Analytical time- and frequency-domain solutions are derived for a constant, axial surface magnetic field after considering the axial symmetry and assuming (i) negligible displacement currents, (ii) linear

* Corresponding author.
E-mail address: dapino.1@osu.edu (M.J. Dapino).

http://dx.doi.org/10.1016/j.jmmm.2015.08.074
0304-8853/

constitutive behavior, (iii) negligible demagnetizing fields, and (iv) uniform stress and electrical conductivity . The solutions are non-dimensionalized and then used to investigate the spatial and frequency dependence of the internal magnetic field and magnetic flux. Unlike the referenced analytical and numerical solutions, these analytical solutions provide design criteria, reveal the relative importance of each material property, and provide expressions for skin depth and cut-off frequency. For nonlinear operating regimes, the derived solutions can be used to assess whether lamination of the magnetoelastic material is necessary.

2. Model development

The general magnetic diffusion equation for magnetoelastic materials is derived from Maxwell's equations and the assumption that displacement currents are negligible

$$-\nabla\left(\nabla\cdot\vec{H}\right)+\nabla^2\vec{H}=\sigma\vec{B}_t=\sigma\mu_0\left(\vec{H}+\vec{M}\right)_t, \tag{1}$$

where the subscript t denotes partial differentiation with respect to time, σ represents the electrical conductivity, μ_0 is the magnetic permeability of free space, and $\vec{H}$, $\vec{B}$, and $\vec{M}$ are the magnetic field strength, magnetic flux density, and magnetization vectors, respectively, which each depend on time t and position. In magnetostrictive materials, $\vec{M}$ depends on the stress vector $\vec{T}$ such that (1) becomes

$$-\nabla\left(\nabla\cdot\vec{H}\right)+\nabla^2\vec{H}-\sigma\left(\left[\mu\right]\vec{H}\right)_t=\sigma\left(\left[d*\right]\vec{T}\right)_t, \tag{2}$$

where $[\mu]$ and $[d*]$ denote the magnetic field- and stress-dependent magnetic permeability and piezomagnetic coefficient tensors, respectively. In ferromagnetic shape memory alloys, $\vec{M}$ depends on the strain vector $\vec{S}$ such that (1) becomes

$$-\nabla\left(\nabla\cdot\vec{H}\right)+\nabla^2\vec{H}-\sigma\left(\left[\mu\right]\vec{H}\right)_t=\sigma\left(\mu_0\left[e\right]\vec{S}\right)_t, \tag{3}$$

where $[e]$ represents the magnetic field- and strain-dependent coupling coefficient tensor.

For biased operation and sufficiently low amplitude excitation, the constitutive tensors $[\mu]$, $[d*]$, and $[e]$ can be assumed to be constant. If a cylindrical magnetostrictive material or ferromagnetic shape memory alloy is operated in a transducer having a closed magnetic circuit of low reluctance, demagnetizing fields can be neglected and the circuit can be represented as an infinitely long rod subjected to a uniform, axial magnetic field H_{ext} at its surface and an axial, distributed force on its ends. Due to the inhomogeneous internal magnetic field, the rod's stiffness, and therefore the applied stress, will be radially dependent [4]. However, to permit an analytical solution, the stress is assumed to be uniform throughout the rod. Stress uniformity along the axial direction is valid for forcing frequencies sufficiently below mechanical resonance of the rod. Due to this assumption, the rod's mechanical inertia and damping (i.e., structural dynamics) are ignored.

Under the aforementioned assumptions, (2) and (3) simplify to

$$H_{rr}(r,t)+H_r(r,t)/r-\sigma\mu H_t(r,t)=\sigma d*T_t(t), \tag{4}$$

and

$$H_{rr}(r,t)+H_r(r,t)/r-\sigma\mu H_t(r,t)=\sigma\mu_0 eS_t(t), \tag{5}$$

respectively, where r is the radial coordinate, the subscript r denotes partial differentiation with respect to r, and μ, d^*, and e are the 33 components of the respective tensors. Thus, the 1D magnetic diffusion problem for ferromagnetic shape memory alloys is identical to that for magnetostrictive materials if $\mu_0 e$ and $S(t)$ are substituted for d^* and $T(t)$, respectively. Consequently, it is sufficient to only solve (4), which resembles the 1D field-induced magnetic diffusion problem, but with a forcing term. Fig. 1 depicts the general mechanically induced magnetic diffusion problem for axial loading of a magnetoelastic cylinder and the simplified problem that is solved.

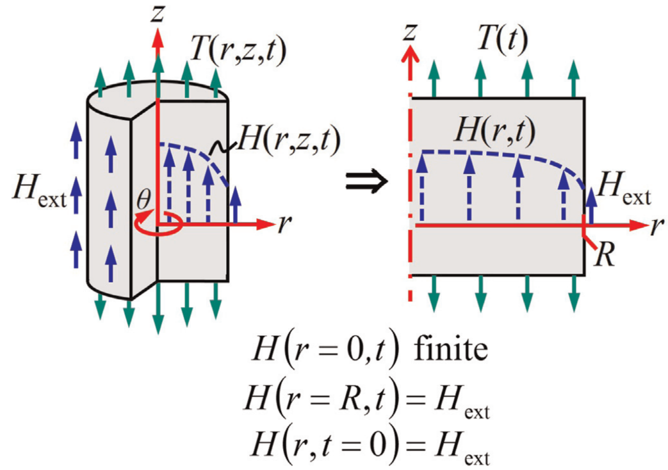

Fig. 1. General mechanically induced magnetic diffusion problem for axial loading (left) and the simplified 1D problem that is solved (right); the magnetic field at the surface of the rod, H_{ext}, is assumed to be uniform and constant in time.

3. 1D time- and frequency-domain solutions

To solve (4), it is convenient to have zero boundary conditions. This is accomplished using the change of variables $\tilde{H}(r,t)=H(r,t)-H_{ext}$, so that the initial boundary value problem is written as

$$\tilde{H}_{rr}(r,t)+\tilde{H}_r(r,t)/r-\sigma\mu\tilde{H}_t(r,t)=\sigma d*T_t(t), \tag{6}$$

$$\tilde{H}(r,t=0)=0, \tag{7}$$

$$\tilde{H}(r=R,t)=0, \tag{8}$$

$$\tilde{H}(r=0,t)\ \text{finite}, \tag{9}$$

where $r=R$ is the surface of the rod. Eqs. (6)–(9) can be written as an inhomogeneous Bessel equation of order zero using the change of variables, $u=\sqrt{\mu\sigma}r$,

$$u^2\tilde{H}_{uu}(u,t)+u\tilde{H}_u(u,t)-u^2\tilde{H}_t(u,t)=\frac{d*}{\mu}u^2T_t(t), \tag{10}$$

$$\tilde{H}\left(\frac{u}{\sqrt{\mu\sigma}},t=0\right)=0, \tag{11}$$

$$\tilde{H}\left(u=\sqrt{\mu\sigma}R,t\right)=0, \tag{12}$$

$$\tilde{H}\left(u=0,t\right)\ \text{finite}, \tag{13}$$