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Relating Mason number to Bingham number in magnetorheological fluids

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ABSTRACT

Magnetorheological (MR) fluids are described using two nondimensional numbers, the Bingham and Mason numbers. The Mason number is the ratio of particle magnetic forces to viscous forces and describes the behavior of MR fluids at the microscopic, particle level scale. At the macroscopic, continuum scale, Bingham number is the ratio of yield stress to viscous stress, and describes the bulk motion of the fluid. If these two nondimensional numbers can be related, then microscopic models can be directly compared to macroscopic results. We show that if microscopic and macroscopic forces are linearly related, then Bingham and Mason number are inversely related, or, alternatively, that the product of the Bingham number and the Mason number is a constant. This relationship is experimentally validated based on measurements of apparent viscosity on a high shear rate, $\dot{\gamma} \approx 10\,000\ \mathrm{s}^{-1}$, Searle cell rheometer. This relationship between Mason number and Bingham number is then used to analyze a Mason number based result, and is also used to inform the MR fluid device design process.

1. Introduction

Magnetorheological (MR) fluid is a fluid composed of micron scale magnetizable particles suspended in a carrier fluid. Upon the application of field, the particles in the fluid align to form chain like structures, and these chains cause the fluid to develop a field dependent yield stress. The primary application of MR fluid has been in MR dampers and MR energy absorbers, where the controllable apparent viscosity allows for a controllable damping force or stroking load, which enables high performance vibration isolation [1] or shock mitigation that can adapt to payload weight and impact severity [2,3].

Models of magnetorheological fluids have typically taken two perspectives: either modeling the MR fluid as a collection of microscopic particles floating in a carrier fluid, or as a bulk fluid continuum. Microscopic modeling of MR fluids focuses on the behavior of the particles [4–6] by examining the formation and destruction of chain structures in the fluid with the goal of predicting yield stress. The primary forces on the particles that govern chain formation are viscous drag of the carrier fluid on the particle and the interparticle magnetic forces. The ratio of particle magnetic forces to viscous forces is known as the Mason number, Mn, [7–9], named after the work of Mason et. al. on the behavior of fluid droplets in the presence of electric field [10]. In the equations of motion, the Mason number is the governing parameter of the shear response of a particle in an MR fluid, and is an essential part of research on dynamic models of chain formation. The Mason number, Mn, also has value in the analysis of experimental data, such as when apparent viscosity is plotted against Mason number, the apparent viscosity curves collapse to a single curve, thereby reducing the dimensionality of a dataset [7,8].

At the bulk scale, one of the idealized descriptions of MR fluids is as a Bingham plastic [11], in which the applied magnetic field additively induces a field controllable yield stress to a Newtonian fluid. The Bingham number, Bi, which is the ratio of yield stress to viscous stress, describes the extent to which the controllable yield stress can exceed the viscous stress (typically $\mathrm{Bi} \gg 1$), and is an essential descriptor of Bingham plastic behavior. The Bingham number can be used to calculate flow rates, flow profiles, and pressure losses in devices using Bingham plastic fluids [12]. In particular, for shear mode MR devices, the Bingham number represents the controllable force ratio [13], and since MR fluids are used for the purpose of generating controllable forces, the Bingham number is an essential and fundamental parameter for the understanding and analysis of MR fluids at the bulk scale.

We seek to relate the Bingham number to the Mason number, two nondimensional numbers that represent fundamental descriptions of the behavior of MR fluid at macroscopic and microscopic scales respectively. In particular, we focus on MR fluids typically used in energy absorbing devices. These MR fluids are typically suspensions of 1–10 μm diameter carbonyl iron particles with solids loading ranging from 20 to 50 volume percent, and

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well described by the Bingham plastic model. By mathematically relating Bingham number to Mason number, we enable microscopic Mason number based analyses to be directly extended to macroscopic or device scale Bingham number based problems. Alternatively, experimental Bingham number based results can be scaled down for comparison to Mason number based particle level analyses.

In this study, the Bingham number and the Mason number are developed, and it is shown that if microscopic forces map linearly to macroscopic forces, then the Bingham number and the Mason number are inversely related, or that the product of Bingham number and Mason number is a constant. This notion is confirmed through measurements of apparent viscosity. We experimentally validate the claim that microscopic and macroscopic forces are linearly related, and that this is akin to assuming that MR fluids are well described by the Bingham plastic model. Finally, the relationship between Mason number and Bingham number is used to examine the experimental relevance of a Mason number based result, as well as how such a relationship would inform the MR fluid and/or device design process.

2. Background

To motivate the usage of these nondimensional numbers, both numbers are derived in the analytical context in which they arise.

2.1. The Bingham number

For device scale analyses, the fluid is treated as a continuum with nonlinear rheological properties. A typical MR fluid shear stress vs. shear rate graph is shown in Fig. 1. These shear stresses for each field strength are typically modeled by the Bingham plastic model,

$$\tau = \tau_y + \eta_{pl}\dot{\gamma}, \tag{1}$$

which has a plastic viscosity, η_{pl}, and a yield stress, τ_y. The yield stress is magnetic field dependent, and it is typical to assume that η_{pl} is independent of field strength, and equivalent to the off-state viscosity, η_{off}. In MR fluids, η_{pl} is chosen to be the slope of the high shear rate asymptote of the shear stress curve, and τ_y corresponds to the intersection of the high shear rate asymptote with the stress axis at $\dot{\gamma} = 0$.

A typical way in which MR fluid is used in damper design is the shear mode damper [13], where an upper plate moving with velocity, v, and area, A, moves over a stationary lower plate with a gap of d between the two plates. Here, the fluid velocity profile is linear, and the force on the upper plate is

$$F_d = \left(\tau_y + \eta_{pl}\frac{v}{d}\right)A. \tag{2}$$

The force in conventional viscous dampers can be written in the form $F_d = c_0 v$, where c_0 is the damping, and for a Newtonian fluid in shear mode $c_0 = \eta A/d$. For the shear mode MR damper, rearranging into this form yields

$$F_d = \left(\frac{\tau_y d}{\eta_{pl} v} + 1\right)\eta_{pl}\frac{A}{d}v = c_{eq}v, \tag{3}$$

where c_{eq} is the equivalent damping for a fluid with a yield stress. The ratio of equivalent damping to Newtonian damping yields the damping coefficient

$$\frac{c_{eq}}{c_0} = 1 + \frac{\tau_y d}{\eta_{pl} v} = 1 + \mathrm{Bi} \tag{4}$$

which describes the effect that the addition of a yield stress has on damping force. For an MR fluid, where the yield stress is field controllable, this ratio is the controllable force ratio. The term that governs controllability is the Bingham number,

$$\mathrm{Bi} = \frac{\tau_y}{\eta_{pl}\dot{\gamma}_c}, \tag{5}$$

the ratio of magnetic forces (τ_y) to viscous forces $\left(\eta_{pl}\dot{\gamma}_c\right)$ in the fluid, where $\dot{\gamma}_c$ is the characteristic shear rate of the system, which for a shear mode damper is $\dot{\gamma}_c = v/d$. Since the purpose of MR fluids is to generate a field controllable force, and the Bingham number represents the controllable force ratio of an MR device, it is clear that Bingham number is a fundamental representation of the behavior of MR fluids. In more complicated geometries, such as in pipe flow, the Bingham number becomes an essential intermediate quantity in the determination of the flow rate, flow profile, and controllable force output of an MR fluid device [12]. But at the fluid level, the Bingham number is a descriptive, empirical quantity, and does not tell us anything about what causes the MR effect, or how a fluid can be modified to improve its performance.

2.2. Mason number

Modeling MR fluid at the particle level allows us to develop predictive models of fluid behavior, providing insight into the chain formation that underlies the MR effect. At the microscopic scale, MR fluids consist of magnetizable particles suspended in a carrier fluid under the influence of an applied magnetic field, H_0. Fig. 2 contains a diagram of two interacting particles under shear and applied magnetic field. Typically, these are spherical carbonyl iron particles with diameter $\sigma = 1 - 10\ \mu$m. The particles are

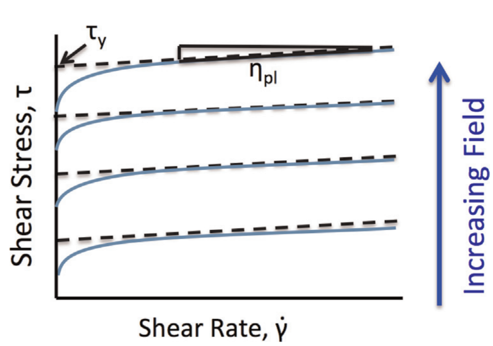

Fig. 1. Idealized rheogram or shear stress vs. shear rate diagram for an MR fluid.

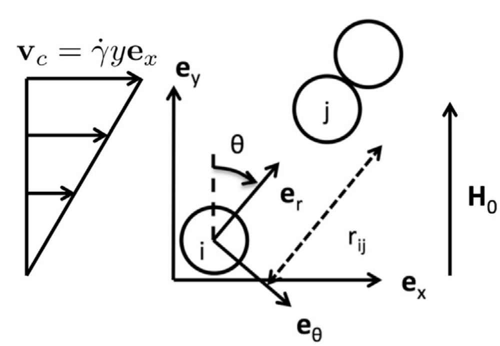

Fig. 2. Diagram of two particles in a shearing fluid.