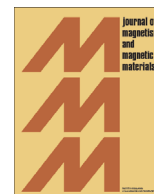




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## Interaction of magnetic field in flow of Maxwell nanofluid with convective effect

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## ABSTRACT

Magnetohydrodynamic (MHD) three-dimensional flow of Maxwell nanofluid subject to the convective boundary condition is investigated. The flow is generated by a bidirectional stretching surface. Thermophoresis and Brownian motion effects are present. Fluid is electrically conducted in the presence of a constant applied magnetic field. Unlike the previous cases even in the absence of nanoparticles, the correct formulation for the flow of Maxwell fluid in the presence of a magnetic field is established. Newly proposed boundary condition with the zero nanoparticles mass flux at the boundary is employed. The governing nonlinear boundary layer equations through appropriate transformations are reduced in the nonlinear ordinary differential system. The resulting nonlinear system has been solved for the velocities, temperature and nanoparticles concentration distributions. Convergence of the constructed solutions is verified. Effects of emerging parameters on the temperature and nanoparticles concentration are plotted and discussed. Numerical values of local Nusselt number are computed and analyzed. It is observed that the effects of magnetic parameter and the Biot number on the temperature and nanoparticles concentration are quite similar. Both the temperature and nanoparticles concentration are enhanced for the increasing value of magnetic parameter and Biot number.

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## 1. Introduction

The investigations on non-Newtonian fluids are remarkably enhanced during the past few decades because of their practical implications in technology and industrial processes. Many of the materials in our daily life include apple sauce, sugar solution, muds, chyme, soaps, emulsion, shampoos, blood at low shear rate etc. exhibits the characteristics of non-Newtonian fluids. In the literature, there is no single relation that characterizes all the properties of non-Newtonian fluids which characterize all the properties of such materials. Many models of non-Newtonian fluids are developed by the researchers in the past. Among these models, Maxwell fluid is a simplest subclass of rate type non-Newtonian fluids. This model is widely used to explore the effects of stress relaxation. The involvement of stress relaxation in the stress tensor of Maxwell fluid makes it highly nonlinear and complicated in comparison to Newtonian fluid. Maxwell fluid model reduced into the simple Navier–Stokes relation when extra

stress time is zero. The boundary layer flows of viscoelastic non-Newtonian fluids have been widely used in engineering technology and industrial applications. Such flows commonly involved in power engineering and food engineering, petroleum production, polymer solutions and in polymer melt, the cooling of a metallic plate in a cooling bath, drawing on plastic films and many others. Abundant studies on this topic exist in the literature, but few interesting and recent studies can be seen in the Refs. [1–8].

Nowadays, the cooling of electronic devices is the major industrial requirements due to the fast technology, but the low thermal conductivity rate of ordinary base fluids includes water, ethylene glycol and oil is the basic limitation. To overcome on such limitation, the nanoscale solid particles are submerged into host fluids which change the thermophysical characteristics of these fluids and enhanced the heat transfer rate dramatically. Choi [9] was the first who identified this colloidal suspension. The recent developments in nanofluids and their mathematical modeling, play vital role in industrial and nanotechnology. The nanofluids are used in the applications such as cooling of electronics, heat exchanger, nuclear reactor safety, hyperthermia, biomedicine, engine cooling, vehicle thermal management and many others. Further the magneto nanofluids are useful in the manufacturing processes

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of industries and biomedicine applications. Examples include in gastric medications, biomaterials for wound treatment, sterilized devices, etc. The magneto nanoparticles can be employed in the elimination of tumors with hyperthermia, targeted drug release and for magnetic resonance imaging. A bulk of research articles on nanofluids is available in the literature in which few can be seen in the Refs. [10–20].

This paper emphasizes on the three-dimensional boundary layer flow of Maxwell fluid induced by a bidirectional stretching surface. Thermophoresis and Brownian motion effects are encountered in the energy and mass species expressions. We considered the thermal convective [21,22] and zero nanoparticles mass flux conditions at the boundaries. The zero nanoparticles mass flux condition was first introduced by Kuznetsov and Nield [15] for two-dimensional boundary layer flows. Here we used this condition for the three-dimensional boundary layer flow of Maxwell nanofluid. The governing nonlinear ordinary differential equations are solved via homotopy analysis method [23–30] The obtained results are sketched and discussed in detail. The values of local Nusselt number are tabulated and examined.

### 2. Mathematical modeling

Consider the steady three-dimensional flow of an incompressible Maxwell nanofluid over a bidirectional stretching surface. Fluid is considered electrically conducting in the presence of constant magnetic field  $B_0$  applied in the z-direction. The Hall and electric field effects are ignored. The induced magnetic field is not considered for a small magnetic Reynolds number. Thermophoresis and Brownian motion effects are taken into account. The temperature at the surface is controlled by a convective heating process which is characterized by the heat transfer coefficient  $h_f$  and temperature of the hot fluid  $T_f$  below the surface. The boundary layer expressions governing the conservations of mass, momentum, energy and nanoparticles concentration are

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0, \tag{1}$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} + \lambda_1 \left( \begin{aligned} &u^2 \frac{\partial^2 u}{\partial x^2} + v^2 \frac{\partial^2 u}{\partial y^2} + w^2 \frac{\partial^2 u}{\partial z^2} \\ &+ 2uv \frac{\partial^2 u}{\partial x \partial y} + 2vw \frac{\partial^2 u}{\partial y \partial z} + 2uw \frac{\partial^2 u}{\partial x \partial z} \end{aligned} \right) = \nu \frac{\partial^2 u}{\partial z^2} - \frac{\sigma B_0^2}{\rho_f} \left( u + \lambda_1 w \frac{\partial u}{\partial z} \right), \tag{2}$$

$$u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + w \frac{\partial v}{\partial z} + \lambda_1 \left( \begin{aligned} &u^2 \frac{\partial^2 v}{\partial x^2} + v^2 \frac{\partial^2 v}{\partial y^2} + w^2 \frac{\partial^2 v}{\partial z^2} \\ &+ 2uv \frac{\partial^2 v}{\partial x \partial y} + 2vw \frac{\partial^2 v}{\partial y \partial z} + 2uw \frac{\partial^2 v}{\partial x \partial z} \end{aligned} \right) = \nu \frac{\partial^2 v}{\partial z^2} - \frac{\sigma B_0^2}{\rho_f} \left( v + \lambda_1 w \frac{\partial v}{\partial z} \right), \tag{3}$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} + w \frac{\partial T}{\partial z} = \alpha \frac{\partial^2 T}{\partial z^2} + \frac{(\rho c)_p}{(\rho c)_f} \left( D_B \left( \frac{\partial T}{\partial z} \frac{\partial C}{\partial z} \right) + \frac{D_T}{T_\infty} \left( \frac{\partial T}{\partial z} \right)^2 \right), \tag{4}$$

$$u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} + w \frac{\partial C}{\partial z} = D_B \left( \frac{\partial^2 C}{\partial z^2} \right) + \frac{D_T}{T_\infty} \left( \frac{\partial^2 T}{\partial z^2} \right). \tag{5}$$

The boundary conditions in the present problem are

$$u = ax, v = by, w = 0, -k \frac{\partial T}{\partial z} = h_f (T_f - T),$$

$$D_B \frac{\partial C}{\partial z} + \frac{D_T}{T_\infty} \frac{\partial T}{\partial z} = 0 \text{ at } z = 0, \tag{6}$$

$$u \rightarrow 0, v \rightarrow 0, T \rightarrow T_\infty, C \rightarrow C_\infty \text{ as } z \rightarrow \infty, \tag{7}$$

In above expressions  $u, v$  and  $w$  are the velocity components in the  $x-, y-$  and  $z-$ directions respectively,  $\lambda_1$  the relaxation time,  $\nu (= \mu / \rho_f)$  the kinematic viscosity,  $\mu$  the dynamic viscosity,  $\rho_f$  the density of base fluid,  $\sigma$  the electrical conductivity,  $T$  the temperature,  $\alpha = k / (\rho c)_f$  the thermal diffusivity of the fluid,  $k$  the thermal conductivity,  $(\rho c)_f$  the heat capacity of the fluid,  $(\rho c)_p$  the effective heat capacity of nanoparticles,  $D_B$  the Brownian diffusion coefficient,  $C$  the nanoparticles concentration,  $D_T$  the thermophoretic diffusion coefficient,  $T_\infty$  the temperature far away from the surface and  $C_\infty$  the nanoparticles concentration far away from the surface.

Using the following transformations

$$u = axf'(\eta), v = ayg'(\eta), w = -(\alpha\nu)^{1/2}(f(\eta) + g(\eta)),$$

$$\theta(\eta) = \frac{T - T_\infty}{T_f - T_\infty}, \varphi(\eta) = \frac{C}{C_\infty}, \eta = \left(\frac{a}{\nu}\right)^{1/2} z. \tag{8}$$

Eq. (1) is automatically satisfied and Eqs. (2)–(7) have the following forms

$$f''' + (M^2\beta + 1)(f + g)f'' - f'^2 + \beta(2(f + g)f'f'' - (f + g)^2f''') - M^2f' = 0, \tag{9}$$

$$g''' + (M^2\beta + 1)(f + g)g'' - g'^2 + \beta(2(f + g)g'g'' - (f + g)^2g''') - M^2g' = 0, \tag{10}$$

$$\theta'' + Pr(f + g)\theta' + Nb\theta'\varphi' + Nt\theta'^2 = 0, \tag{11}$$

$$\varphi'' + LePr(f + g)\varphi' + \frac{Nt}{Nb}\theta'' = 0, \tag{12}$$

$$f = 0, g = 0, f' = 1, g' = c, \theta' = -\gamma(1 - \theta(0)),$$

$$Nb\varphi' + Nt\theta' = 0 \text{ at } \eta = 0, \tag{13}$$

$$f' \rightarrow 0, g' \rightarrow 0, \theta \rightarrow 0, \varphi \rightarrow 0 \text{ as } \eta \rightarrow \infty. \tag{14}$$

where  $\beta$  is the Deborah number,  $M$  is the magnetic parameter,  $c$  is the ratio of stretching rates,  $Pr$  is the Prandtl number,  $Nb$  is the Brownian motion parameter,  $Nt$  is the thermophoresis parameter,  $\gamma$  is the Biot number,  $Le$  is the Lewis number and prime stands for differentiation with respect to  $\eta$ . These parameters can be expressed by the following definitions:

$$\beta = \lambda_1 a, M^2 = \frac{\sigma B_0^2}{\rho_f a}, c = \frac{b}{a}, Pr = \frac{\nu}{\alpha}, Nb = \frac{(\rho c)_p D_B C_\infty}{(\rho c)_f \nu},$$

$$Nt = \frac{(\rho c)_p D_T (T_f - T_\infty)}{(\rho c)_f \nu T_\infty}, \gamma = \frac{h_f}{k} \sqrt{\frac{\nu}{a}}, Le = \frac{\alpha}{D_B}. \tag{15}$$

The local Nusselt number  $Nu_x$  is defined as

$$Nu_x = - \frac{x}{(T_w - T_\infty)} \frac{\partial T}{\partial z} \Big|_{z=0} = - (Re_x)^{1/2} \theta'(0). \tag{16}$$

It is noted that the dimensionless mass flux represented by a Sherwood number  $Sh_x$  is now identically zero and  $Re_x = ux/\nu$  is the local Reynolds number.

### 3. Series solutions

The initial guesses and linear operators for homotopic solutions are

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