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The efficiency and the demagnetization field of a general Halbach cylinder

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ABSTRACT

The maximum magnetic efficiency of a general multipole Halbach cylinder of order p is found as function of p. The efficiency is shown to decrease for increasing absolute value of p. The optimal ratio between the inner and outer radius, i.e. the ratio resulting in the most efficient design, is also found as function of p and is shown to tend towards smaller and smaller magnet sizes. Finally, the demagnetizing field in a general p-Halbach cylinder is calculated, and it is shown that demagnetization is largest either at $\cos 2p\phi = 1$ or $\cos 2p\phi = -1$. For the common case of a $p=1$ Halbach cylinder the maximum values of the demagnetizing field are either at $\phi = 0, \pi$ at the outer radius, where the field is always equal to the remanence, or at $\phi = \pm\pi/2$ at the inner radius, where it is the magnitude of the field in the bore. Thus to avoid demagnetization the coercivity of the magnets must be larger than these values.

1. Introduction

Generating a powerful magnetic field with a permanent magnet assembly is important in a number of applications. It is also often of importance that the magnetic field is generated using the least amount of magnet material possible. A common magnet design used to generate such powerful magnetic fields is the Halbach cylinder, which has been used in a number of applications, such as nuclear magnetic resonance (NMR) equipment [1,2], accelerator magnets [3,4], magnetic refrigeration devices [5,6] and medical applications [7]. The Halbach cylinder is a hollow cylinder made of a ferromagnetic material with a remanent flux density which in cylindrical coordinates is given by

$$B_{\mathrm{rem},r} = B_{\mathrm{rem}} \cos p\phi$$
$$B_{\mathrm{rem},\phi} = B_{\mathrm{rem}} \sin p\phi, \quad (1)$$

where B_{rem} is the magnitude of the remanent flux density and p is an integer [8,9]. For p positive a field inside the cylinder bore is generated, which for the important case of $p=1$ is spatially uniform. There is zero field outside the cylinder. For p negative the Halbach cylinder creates a field on its outside, while the inside field becomes zero. Usually, as will also be the case here, a two dimensional problem is considered. This is a good approximation if the radius of the cylinder is smaller than its length. The magnetic field distribution for such a Halbach cylinder of infinite length [10–14] as well as for finite length [15–18] has previously been investigated in detail. However, the efficiency of Halbach cylinders has not been considered for a general p-Halbach cylinder.

The plan of the paper is as follows: First we calculate the efficiency of a p-Halbach cylinder and thereafter we consider the demagnetization field internally in the Halbach cylinder and its possible influence on the performance and efficiency of the Halbach cylinder. Finally, we discuss the implications of our findings.

We consider the field from a p-Halbach in general. The geometry is as shown in Fig. 1 for the case of an ‘interior’ Halbach (positive p); for an ‘exterior’ Halbach (corresponding to negative p) the field is generated on the outside of the cylinder. We assume that the permanent magnets are perfectly linear and with a relative permeability of $\mu_r = 1$, which is a good approximation for NdFeB magnets.

2. Efficiency of a Halbach cylinder

The object of a permanent magnet array is to generate a magnetic field of given characteristics in a given volume. Many different magnet configurations can in principle produce the same magnetic field, and thus the question arises of how to do it most efficiently. Jensen and Abele [19] proposed a general figure of

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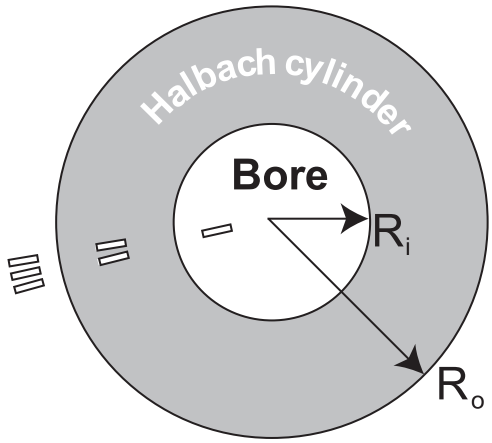

Fig. 1. The Halbach cylinder geometry for a p-Halbach ($p > 0$). The different radii and regions, I–III, have been indicated.

merit, M^*, to characterize the efficiency of a given magnet design:

$$M^* = \frac{\int_{V_{\text{field}}} \|\mathbf{B}\|^2\, dV}{\int_{V_{\text{mag}}} \|\mathbf{B}_{\text{rem}}\|^2\, dV}, \tag{2}$$

where V_{field} is the volume of the region where the magnetic field is created and V_{mag} is the volume of the magnets. The figure of merit is the ratio of the energy stored in the field region to the maximum amount of magnetic energy available in the magnetic material. It can be shown that the maximum value of M^* is 0.25 [19].

We here consider the efficiency of a p-Halbach cylinder. While the efficiency of a $p=1$ Halbach cylinder have been considered several times [18,20,21], the efficiency of a general $p \neq 1$ Halbach cylinder have not been considered in the literature, although the calculation is straightforward. The efficiency can be found using the analytical solution to the field equations for the magnetic field for a general p-Halbach cylinder. These are given in Bjørk et al. [14], where the field interior to the cylinder is given as

$$\begin{pmatrix} B_r(r,\phi) \\ B_\phi(r,\phi) \end{pmatrix} = \begin{cases} B_{\text{rem}} \dfrac{p}{p-1}\left[1 - \left(\dfrac{R_i}{R_o}\right)^{p-1}\right]\left(\dfrac{r}{R_i}\right)^{p-1}\begin{pmatrix} \cos p\phi \\ -\sin p\phi \end{pmatrix}, & p > 1 \\ B_{\text{rem}} \ln\left(\dfrac{R_o}{R_i}\right)\begin{pmatrix} \cos\phi \\ -\sin\phi \end{pmatrix}, & p = 1. \end{cases} \tag{3}$$

Here R_o is the outer radius of the Halbach and R_i the inner radius, as shown in Fig. 1.

For an exterior Halbach ($p \leq -1$) the field outside the Halbach is

$$\begin{pmatrix} B_r(r,\phi) \\ B_\phi(r,\phi) \end{pmatrix} = B_{\text{rem}} \frac{p}{p-1}\left[1 - \left(\frac{R_o}{R_i}\right)^{p-1}\right]\left(\frac{r}{R_o}\right)^{p-1}\begin{pmatrix} \cos p\phi \\ -\sin p\phi \end{pmatrix}. \tag{4}$$

Performing the integral in Eq. (2) for an internal and external Halbach cylinder yields

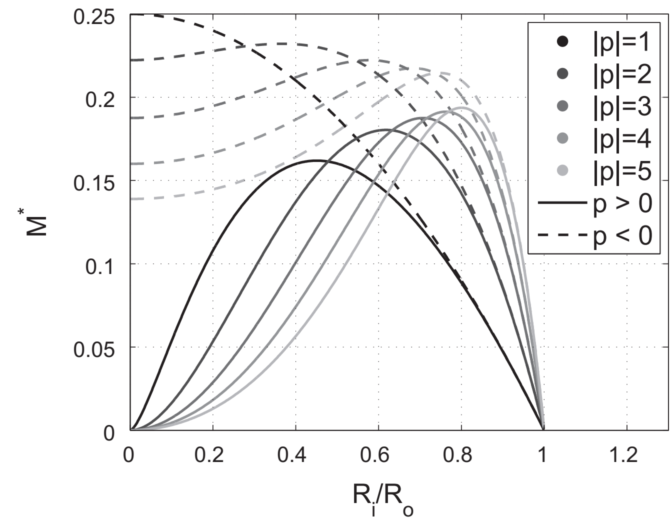

Fig. 2. The efficiency, M^*, as a function of the ratio of the inner and outer radius for Halbach cylinders with $p = -5$ to $p=5$.

$$M^* = \begin{cases} \dfrac{(R_i/R_o)^2}{1-(R_i/R_o)^2}\dfrac{p}{(1-p)^2}\left(1 - \left(\dfrac{R_i}{R_o}\right)^{p-1}\right)^2, & p > 1 \\ \ln\left(\dfrac{R_o}{R_i}\right)^2 \dfrac{(R_i/R_o)^2}{1-(R_i/R_o)^2}, & p = 1 \\ -\left(\dfrac{R_i}{R_o}\right)^{-2p}\dfrac{(R_i/R_o)^2}{1-(R_i/R_o)^2}\dfrac{p}{(1-p)^2}\left(1 - \left(\dfrac{R_i}{R_o}\right)^{p-1}\right)^2 & p \leq -1 \end{cases} \tag{5}$$

The efficiency as a function of the ratio between the radii is plotted in Fig. 2 for $p = -5$ to $p=5$.

It is of interest to determine the optimal efficiency possible for a general Halbach cylinder and the ratio of the inner and outer radius at which this occurs. This can be done by taking the derivative of M^* (Eq. (5)) with respect to the ratio of the radii and equating it to zero. The resulting equation is, as can be seen below, a polynomial equation of order $p+1$, which does not in general have a closed form solution for $p > 3$:

$$\begin{aligned} &1 - p\left(\frac{R_i}{R_o}\right)^{p-1} + (p-1)\left(\frac{R_i}{R_o}\right)^{p+1} = 0, \quad p > 1 \\ &-1 + p - p\left(\frac{R_i}{R_o}\right)^2 + \left(\frac{R_i}{R_o}\right)^{p+1} = 0, \quad p \leq -1 \end{aligned} \tag{6}$$

For the case of $p=1$ the solution is given by $R_i/R_o = e^{-W(-2e^{-2})/2-1}$ where W is the Lambert W function. The argument of the Lambert W function is greater than $-1/e$, which means that the function is single-valued. Evaluated numerically this corresponds to the well-known ratio of $R_o/R_i = 2.2185$, although only the numerical solution has been given previously in the literature [20,21]. For other values of p Eq. (6) can be solved numerically. The solution, i.e. the highest efficiency and corresponding ratio of the radii, is shown in Figs. 3 and 4 as a function of p. In order to verify the analytical results for the field, the value of M^* for the optimal ratio of the radii was also calculated numerically using Comsol Multiphysics finite element software (FEM) and was found to match the analytical value.

For the case of $p = -1$, the Halbach cylinder is simply a uniformly magnetized circle or infinite rod. For this system the