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# Robust solution procedure for the discrete energy-averaged model on the calculation of 3D hysteretic magnetization and magnetostriction of iron–gallium alloys

H. Tari, J.J. Scheidler, M.J. Dapino *,[1]

*Department of Mechanical and Aerospace Engineering, The Ohio State University, Columbus, Ohio 43210, USA*

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ABSTRACT

A reformulation of the Discrete Energy-Averaged model for the calculation of 3D hysteretic magnetization and magnetostriction of iron-gallium (Galfenol) alloys is presented in this paper. An analytical solution procedure based on an eigenvalue decomposition is developed. This procedure avoids the singularities present in the existing approximate solution by offering multiple local minimum energy directions for each easy crystallographic direction. This improved robustness is crucial for use in finite element codes. Analytical simplifications of the 3D model to 2D and 1D applications are also presented. In particular, the 1D model requires calculation for only one easy direction, while all six easy directions must be considered for general applications. Compared to the approximate solution procedure, it is shown that the resulting robustness comes at no expense for 1D applications, but requires almost twice the computational effort for 3D applications. To find model parameters, we employ the average of the hysteretic data, rather than anhysteretic curves, which would require additional measurements. An efficient optimization routine is developed that retains the dimensionality of the prior art. The routine decouples the parameters into exclusive sets, some of which are found directly through a fast preprocessing step to improve accuracy and computational efficiency. The effectiveness of the model is verified by comparison with existing measurement data.



## 1. Introduction

Magnetostrictive materials undergo dimensional changes when exposed to a magnetic field and exhibit magnetization changes when subjected to external stress. Design and optimization of magnetostrictive systems using finite element techniques require constitutive models that are robust and valid for arbitrary magnetic field and stress inputs. The robustness of a constitutive model is dictated by the complexity of the material behavior.

Galfenol (FeGa) is a magnetostrictive material that was developed at the Naval Surface Warfare Center [1]. Galfenol is well suited for actuation and sensing applications as it possesses high tensile strength (~500 MPa), demonstrates moderate magnetostriction (~350 ppm) under very low magnetic fields (~8 kA/m), and exhibits limited temperature dependence ([2]) in its active properties between −20 and 80 °C. In addition, Galfenol has very low hysteresis, a high Curie temperature (675 °C), and corrosion resistance in aqueous environments [2,3]. Galfenol further benefits from its moderate machinability and ductility, and its relatively inexpensive constituent materials. A detailed review of these alloys was presented by Atulasimha and Flatau [2].

Due to nonlinear response of magnetostrictive materials, modeling their behavior for arbitrary stress and magnetic field inputs is challenging. A wide range of models have been proposed. At one extreme, a phenomenological approach fits a curve or surface to the measurement data, which provides efficiency but ignores the underlying physics. At the other extreme, micromagnetic models consider all known energies and are very accurate. Macroscopic models use an intermediate approach by relating the macroscopic response of the material to simplified descriptions of the microscopic behavior. Macroscopic models therefore strike a balance between efficiency, accuracy, and predictive capability. The classical macroscopic models are the Preisach model [4], the Globus model [5], the Jiles–Atherton model [6], and the Stoner–Wohlfarth model [7]. Liorzou et al. [8] compare these models in detail.

* Corresponding author.
*E-mail addresses:* tari.1@osu.edu (H. Tari), scheidler.8@osu.edu (J.J. Scheidler), dapino.1@osu.edu (M.J. Dapino).
[1] Author to whom correspondence should be addressed.

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Armstrong [9] proposed an incremental model for magnetoelastic hysteresis in which the bulk magnetization and magnetostriction are the expected values of a large collection of non-interacting magnetic moments. The probability density function is a Boltzmann distribution, where minimum energy orientations are more probable. The Armstrong model is computationally inefficient, as it searches for global energy minima. Atulasimha et al. [10] improved efficiency by only considering 98 fixed orientations. Evans and Dapino [11] proposed a discrete energy-averaged (DEA) model, which greatly improved efficiency while maintaining accuracy by solving for the local energy minimum about each of Galfenol's six easy crystallographic directions. The speed of this model was further increased by Chakrabarti [13], whose model is the starting point of our development.

The DEA model calculates the magnetization unit directions that minimize a Gibbs free energy defined locally about the six easy directions. However, the resulting minimization problem is not amenable to an explicit analytical solution. To alleviate this shortcoming, Evans and Dapino [11] linearized the normalization constraint about each easy direction, which is valid for small rotations of the magnetic moments about the easy directions. This novel treatment resulted in an explicit approximate solution, whose error is minimal since moments that have rotated far from the easy axes are more energetic, and thus less probable [12]. Nonetheless, the approximate magnetization directions may significantly violate the unity norm constraint for large inputs. Post-normalization of the directions was proposed by Chakrabarti and Dapino [14], but the resulting directions can still deviate from the true energy minima for generic 3D inputs. Additionally, this solution procedure is prone to singularities, which could burden the computation, especially when the method is integrated into finite element solvers.

The primary objective of this paper is to develop a robust solution procedure for the DEA model that avoids singularities. First, we improve the DEA model by incorporating a more thorough expression for the magnetostriction in cubic ferromagnets and by formulating the model based on the average of hysteretic data, which precludes the need for additional anhysteretic data. Then, an analytical solution of the resulting model is derived that exactly solves the constrained energy minimization. Depending on the dimension of the application, the solution procedure offers multiple solutions for each easy direction, which allows it to circumvent singularities and completely reveal the material behavior for arbitrary magnetic field and stress inputs. By having an analytical solution, the model is significantly simplified for 2D and 1D applications. A novel parameter optimization routine is developed, which decouples the model parameters into two sets. One set is quickly calculated through a preprocessing step, while the other is determined through a sophisticated constrained minimization. It is shown that the increased robustness of the proposed model comes at no expense for 1D applications, but requires almost twice the computation time for generic 3D applications. The model is validated through comparison with existing measurements and the former model.

The rest of the paper is organized as follows. A brief review of the DEA model and the existing approximate solution is shown in Section 2. Section 3.1 presents a reformulation of the DEA model and the analytical solution procedure. Section 4 gives the reduced formulation of the model for 2D and 1D applications. Parameter optimization is discussed in Section 5, which is followed by model validation and the conclusions.

## 2. Review of 3D magnetostriction ($\lambda$) and magnetization (M) calculation

Evans and Dapino [11] proposed a discrete energy-averaged (DEA) model that computes the magnetization directions of mesoscopic magnetic domains by minimizing the Gibbs free energy that is defined locally about each easy crystallographic direction. The Gibbs free energy in the vicinity of the $k$th easy direction is composed of magnetocrystalline (anisotropy), magnetoelastic (magnetomechanical coupling), and magnetic field (Zeeman) energies. The minimization procedure applied to Evans and Dapino's formulation requires as many matrix inversions as the number of easy directions. To reduce the number of matrix inversions to one, Chakrabarti [13] slightly modified the anisotropy energy. Accordingly, the Gibbs free energy can be written in matrix notation as

$$\overset{k}{G} = \tfrac{1}{2}\,\overset{k}{\mathbf{m}}\cdot\overline{\mathbf{K}}\overset{k}{\mathbf{m}} - \overset{k}{\mathbf{b}}\cdot\overset{k}{\mathbf{m}} + \tfrac{1}{2}K + \overset{k}{K}_0, \tag{1}$$

where $K$ and $\overset{k}{K}_0$ are anisotropy energy constants; $\overset{k}{\mathbf{m}} = [\overset{k}{m}_1; \overset{k}{m}_2; \overset{k}{m}_3]$ is the magnetization direction having unit magnitude; $k$ takes values $\pm1, \ldots, \pm r/2$; and $r$ is the number of easy crystallographic directions (the $\langle 100\rangle$ family of six directions for Galfenol). The magnetic stiffness matrix $\overline{\mathbf{K}}$ and magnetic force vector $\mathbf{b}$ along the $k$th easy direction are given by

$$\overline{\mathbf{K}} = K\mathbf{I} - 3\begin{bmatrix} \lambda_{100}T_1 & \lambda_{111}T_4 & \lambda_{111}T_6 \\ \lambda_{111}T_4 & \lambda_{100}T_2 & \lambda_{111}T_5 \\ \lambda_{111}T_6 & \lambda_{111}T_5 & \lambda_{100}T_3 \end{bmatrix} \tag{2}$$

and

$$\overset{k}{\mathbf{b}} = K\overset{k}{\mathbf{c}} + \mu_0 M_s\mathbf{H}, \tag{3}$$

respectively, where $\mathbf{I}$ is the $3\times3$ identity matrix; $\lambda_{100}$ and $\lambda_{111}$ are magnetostriction constants; $\mu_0$ and $M_s$ are, respectively, the vacuum permeability and saturation magnetization; $\mathbf{H} = [H_1; H_2; H_3]$ is the magnetic field vector; and $\mathbf{T} = [T_1; T_2; T_3; T_4; T_5; T_6]$ stands for the stress tensor written in contracted vector notation, where $T_1 = T_{11}$, $T_2 = T_{22}$, $T_3 = T_{33}$, $T_4 = T_{12}$, $T_5 = T_{23}$, and $T_6 = T_{13}$.

The macroscopic 3D magnetostriction $\boldsymbol{\lambda}$ and magnetization $\mathbf{M}$ are defined as weighted sums of the response due to the $r$ minimum energy directions,

$$\boldsymbol{\lambda} \triangleq \overline{\boldsymbol{\lambda}} = \sum_{k=\pm1}^{\pm r/2} \overset{k}{\xi}_{an}\overset{k}{\boldsymbol{\lambda}}, \tag{4}$$

$$\mathbf{M} \triangleq \overline{\mathbf{M}} = M_s \sum_{k=\pm1}^{\pm r/2} \overset{k}{\xi}_{an}\overset{k}{\mathbf{m}}, \tag{5}$$

where $\overset{k}{\xi}_{an}$ and $\overset{k}{\boldsymbol{\lambda}}$ denote, respectively, the bulk anhysteretic volume fraction and the magnetostriction tensor written in vector notation for the $k$th domain. Letting $\Omega$ be a smoothing factor, the former is calculated as a Boltzmann-type, energy-weighted average as

$$\overset{k}{\xi}_{an} = \exp\left(-\frac{\overset{k}{G}}{\Omega}\right)\Bigg/\sum_{j=\pm1}^{\pm r/2}\exp\left(\frac{\overset{j}{G}}{\Omega}\right), \tag{6}$$

and the magnetostriction in tensor notation is given as

$$\begin{aligned} \overset{k}{\lambda}_{uu} &= \frac{3}{2}\lambda_{100}\overset{k}{m}_u^{\,2} \\ \overset{k}{\lambda}_{uv} &= 3\lambda_{111}\overset{k}{m}_u\overset{k}{m}_v, \quad u \neq v \end{aligned} \qquad , \quad u, v \in \{1, 2, 3\}. \tag{7}$$

### 2.1. Calculation of $\overset{k}{\mathbf{m}}$ (approximate solution)

The application of an external magnetic field or stress changes