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Magnetohydrodynamic three-dimensional flow of viscoelastic nanofluid in the presence of nonlinear thermal radiation

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ABSTRACT

Magnetohydrodynamic (MHD) three-dimensional flow of couple stress nanofluid in the presence of thermophoresis and Brownian motion effects is analyzed. Energy equation subject to nonlinear thermal radiation is taken into account. The flow is generated by a bidirectional stretching surface. Fluid is electrically conducting in the presence of a constant applied magnetic field. The induced magnetic field is neglected for a small magnetic Reynolds number. Mathematical formulation is performed using boundary layer analysis. Newly proposed boundary condition requiring zero nanoparticle mass flux is employed. The governing nonlinear mathematical problems are first converted into dimensionless expressions and then solved for the series solutions of velocities, temperature and nanoparticles concentration. Convergence of the constructed solutions is verified. Effects of emerging parameters on the temperature and nanoparticles concentration are plotted and discussed. Skin friction coefficients and Nusselt number are also computed and analyzed. It is found that the thermal boundary layer thickness is an increasing function of radiative effect.

1. Introduction

Sustainable energy generation at present is a critical issue in human society development. Shortage of global energy and environmental pollution issues arise widely due to the fast development of human society during the past few decades. Researchers in the field are trying to develop new energy technologies and explore new energy sources for the sustainable energy. Solar energy is regarded one of the best sources of renewable energy which creates energy for billions of years. Nanomaterials are introduced as a new energy material. These particles are undoubtedly capable to absorb the incident radiations. Solar power is considered important to produce electricity and heat from nature. Besides this the thermal radiative transport has great importance in many engineering applications like solar power collectors, heating and cooling chambers, large open water reservoirs and many other industrial and environmental processes.

The suspension of ultrafine nanoparticles in a base fluid is known as nanofluid. Nanofluid is a new variety of heat transfer fluids. To meet the cooling rate requirements in the industry, the thermal performance of ordinary heat transfer fluids is not suitable. The concept of insertion of nanometer sized metallic particles in fluid leads to an increase in the thermal conductivity of base liquids. In fact, the thermal conductivity of the fluid is dramatically enhanced due to the presence of nanoparticles. Such fluids are very interesting alternative in micro-electromechanical systems, nuclear reactors, electronic cooling equipment, transportation, heating and cooling process of energy conversion, etc. Choi [1] introduced the term nanofluid and showed that the insertion of nanoparticles in a base fluid enhances the thermal properties of base liquids. Buongiorno [2] developed a mathematical model to explore the thermal properties of base fluids. Here Brownian motion and thermophoresis are utilized to enhance the thermal properties of base liquids. After that the researchers investigated the flow of nanofluid under different conditions and different types of nanoparticles. Khan and Pop [3] used the Buongiorno [2] model to investigate the boundary layer flow of nanofluid past a stretching sheet. The Keller-Box method is used for the numerical solutions of the modeled differential system. Boundary layer flow of viscous nanofluid past a convectively heated plate was addressed by Makinde and Aziz [4] They showed that the convective boundary condition is more realistic than the constant surface temperature or heat flux conditions. Oztop et al. [5] examined the natural convection flow of nanofluid over a non-isothermal temperature distribution. Thermal and concentration stratification effects in mixed convection boundary layer flow of

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nanofluid over a flat plate were studied by Ibrahim and Makinde [6] Turkyilmazoglu and Pop [7] investigated the unsteady natural convection flow of nanofluid in the presence of thermal radiation over a vertical flat surface. Influence of nanoparticles in Jeffrey-Hamel flow was investigated by Moradi et al. [8] Rashidi et al. [9] carried out a study to discuss the second law of thermodynamics in MHD flow of nanofluid over a rotating porous disk. Zeeshan et al. [10] studied the flow of viscous nanofluid between the concentric cylinders. Sheikholeslami et al. [11] addressed the MHD flow of nanofluid between two horizontal parallel plates in a rotating system. Hayat et al. [12] investigated the mixed convection peristaltic flow of nanofluid in presence of slip and Joule heating effects. Rahman et al. [13] examined the flow of nanofluid due to an exponentially shrinking surface with the second order slip condition. Heat transfer analysis of magnetohydrodynamic flow of nanofluid in a rotating system was discussed by Sheikholeslami et al. [14] Very recently, Kuznetsov and Nield [15] provided the revised model of natural convective boundary layer flow of nanofluid past a vertical plate. In this work they argued that the present model is more realistic than the previous models. This study showed that the nanofluid particle fraction on the boundary is passively rather than actively controlled.

The boundary layer flow induced by a stretching surface has tremendous applications in plastic and metal industries which include manufacturing of plastic and rubber sheets, hot rolling, wire drawing, continuous cooling of fiber spinning, annealing and thinning of copper wires, drawing on stretching sheets through quiescent fluids, boundary layer along a liquid film condensation process and aerodynamic extrusion of plastic films. Several fluids like paints, shampoos, ketchup, paper pulp, apple sauce, certain oils and polymer solutions are the examples of non-Newtonian fluids. A single constitutive relationship cannot explore the diverse characteristics of non-Newtonian materials. Different models have been developed to explain the behavior of non-Newtonian fluids. Among these the couple stress fluids [16–20] have distinct features such as the presence of couple stresses, body couples and non-symmetric stress tensor. The study of couple stress fluids has many applications in industrial processes such as the extrusion of polymer fluids, cooling of the metallic plate in a bath, solidification of liquid crystals and colloidal solutions, etc. These fluids are capable of describing various types of lubricants, blood, suspension fluids and electro rheological fluids.

The objective of the present study is to develop a mathematical model for magnetohydrodynamic (MHD) three-dimensional boundary layer flow of couple stress nanofluid in the presence of nonlinear thermal radiation over a stretching surface. Effects of thermophoresis and Brownian motion are considered. To our knowledge, no such analysis of couple stress fluid is performed in the literature yet. Thus we prefer to utilize here the recent condition of ref. [15] in the three-dimensional flow of couple stress fluid. Similarity variables are employed to convert the nonlinear partial differential system into the ordinary differential system. The resulting strong nonlinear system is computed and analyzed through the series solutions by homotopy analysis method (HAM) [21–26] Convergence analysis of obtained series solutions is made graphically and numerically. The discussion of plots and numerical values with respect to various parameters of interest is arranged. It is hoped that the magneto nanofluids are useful in MHD power generators, petroleum reservoirs, wound treatment, gastric medications, sterilized devices, cancer therapy and to guide the particles up in the bloodstream to a tumor with magnets. A few recent studies regarding magneto nanofluids can be found in the investigations [27–30]

2. Mathematical modeling

Consider the three-dimensional flow of an incompressible couple stress nanofluid. The flow is caused by a bidirectional stretching surface. The fluid is considered electrically conducting in the presence of constant magnetic field B_0 applied in the z−direction. In addition the Hall and electric field effects are ignored. The induced magnetic field is not considered for a small magnetic Reynolds number. Nonlinear thermal radiation, Brownian motion and thermophoresis effects are taken into account. We adopt the Cartesian coordinate system in such a way that x− and y−axes are in the direction of motion and z−axis is normal to the sheet. The sheet at $z = 0$ is stretched in the x− and y−directions with velocities U_w and V_w respectively. The governing boundary layer equations in the present flow analysis are

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0, \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} = \nu\frac{\partial^2 u}{\partial z^2} - \nu'\frac{\partial^4 u}{\partial z^4} - \frac{\sigma B_0^2}{\rho_f}u, \tag{2}$$

$$u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z} = \nu\frac{\partial^2 v}{\partial z^2} - \nu'\frac{\partial^4 v}{\partial z^4} - \frac{\sigma B_0^2}{\rho_f}v, \tag{3}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} + w\frac{\partial T}{\partial z} = \alpha\frac{\partial^2 T}{\partial z^2} - \frac{1}{(\rho c)_f}\frac{\partial q_r}{\partial z} + \frac{(\rho c)_p}{(\rho c)_f}\left(D_B\left(\frac{\partial T}{\partial z}\frac{\partial C}{\partial z}\right) + \frac{D_T}{T_\infty}\left(\frac{\partial T}{\partial z}\right)^2\right), \tag{4}$$

$$u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y} + w\frac{\partial C}{\partial z} = D_B\left(\frac{\partial^2 C}{\partial z^2}\right) + \frac{D_T}{T_\infty}\left(\frac{\partial^2 T}{\partial z^2}\right). \tag{5}$$

The boundary conditions for the present flow analysis are [15]:

$$u = U_w(x) = ax,\quad v = V_w(y) = by,\quad w = 0,\ T = T_w(x),\quad D_B\frac{\partial C}{\partial z} + \frac{D_T}{T_\infty}\frac{\partial T}{\partial z} = 0 \text{at} z = 0, \tag{6}$$

$$u \to 0,\ v \to 0,\ T \to T_\infty,\ C \to C_\infty \text{as} z \to \infty. \tag{7}$$

In above equations u, v and w are the velocity components in the $x-$, $y-$ and $z-$directions respectively, $\nu = \mu/\rho_f$ the kinematic viscosity, μ the dynamic viscosity, ρ_f the density of base fluid, $\nu' = n/\rho_f$ the couple stress viscosity, n the couple stress viscosity parameter, σ the electrical conductivity, T the temperature, $\alpha = k/(\rho c)_f$ the thermal diffusivity of base fluid, k the thermal conductivity, $(\rho c)_f$ the heat capacity of fluid, q_r the nonlinear radiative heat flux, $(\rho c)_p$ the effective heat capacity of nanoparticles, D_B the Brownian diffusion coefficient, C the nanoparticles concentration, D_T the thermophoretic diffusion coefficient, T_w and T_∞ the temperatures of the surface and far away from the surface and C_∞ the nanoparticles concentration far away from the surface. The subscript w denotes the wall condition. Here we assume that the surface stretching velocities and wall temperature are

$$U_w(x) = ax,\ V_w(y) = by,\ T_w(x) = T_\infty + T_0 x, \tag{8}$$

where a, b and T_0 are the positive constants. The nonlinear radiative heat flux q_r via Rosseland's approximation is