Journal of Magnetism and Magnetic Materials 378 (2015) 320–326

Contents lists available at ScienceDirect

Journal of Magnetism and Magnetic Materials

journal homepage: www.elsevier.com/locate/jmmm

Metachronal beating of cilia under the influence of Casson fluid and magnetic field

Noreen Sher Akbar [a,*], Zafar Hayat Khan [b]

[a] DBS&H, CEME, National University of Sciences and Technology, Islamabad, Pakistan
[b] Department of Mathematics, University of Malakand, Chakdara, Dir (Lower), Khyber Pakhtunkhwa, Pakistan

ARTICLE INFO

Article history:
Received 1 September 2014
Received in revised form
16 November 2014
Accepted 17 November 2014
Available online 20 November 2014

Keywords:
Metachronal beating of cilia
Symmetric channel
Casson fluid
Magnetic field

ABSTRACT

Metachronal beating of cilia under the influence of Casson fluid and magnetic field is considered. The model for cilia literature is modelled for the first time. The governing coupled equations are constructed under long wavelength and low Reynold's number approximation. Exact solutions are evaluated for stream function and pressure gradient. The important results in this study are the variation of the Hartmann number M, Casson fluid parameter ζ. The velocity field increases due to the increase in Hartmann number M near the channel walls while velocity field decreases at the center of the channel. Comparative study is also made for Casson fluid with Newtonian fluid.

1. Introduction

A magnetic field is the magnetic influence of electric currents and magnetic materials. The magnetic field at any given point is specified by both a direction and a magnitude. Applied magnetic field effects with fluids can be seen in certain engineering processes like glass manufacturing, Crude Oil refinement, paper production and in some geophysical studies. Mekheimer and Elmaboud [1] studied the effects of magnetic field on peristaltic transport of a viscous fluid in an annulus. They also present the application of an endoscope. In this regard the effect of applied magnetic field in circular tube is discussed by Ebaid [2]. Magnetic field with the peristaltic flow through a porous space with compliant walls is presented by Srinivas and Kothandapani [3]. Combining effects of slip conditions, wall properties and heat transfer on MHD peristaltic transport is debated by Srinivas et al. [4]. The influence of variable viscosity for magneto-hydrodynamic third grade fluid is coated by Ellahi et al. [5]. In another article Ellahi et al. [6] investigated the peristaltic motion of Oldroyd fluid in an asymmetric channel with MHD. Very recently the effects of MHD on Cu–water nanofluid flow and heat transfer by considering Lorentz force are analyzed by Sheikholeslami et al. [7,8]. They modelled the problem and then the analytical solutions of coupled equations are developed by regular perturbation method. Abd-Alla et al. [9,10] presented combined effects of rotation and magnetic field in peristalsis for different non-Newtonian fluids.

Ciliary movement is generated in the axoneme by the unidirectional sliding of the outer doublets of microtubules produced by the adenosine triphosphate (ATP)-energized dynein arms. It can be obtained in the absence of membrane or ciliary matrix and can be reproduced on “models” of cilia whose membrane has been partially or completely destroyed by detergents. Thus, isolated cilia or flagella, or “models” of whole ciliates (for instance, Paramecium), gently treated with Triton X-100, can be reactivated and are able to swim normally when they are bathed in an appropriate medium whose main components are the specific substrates for dynein see Refs. [11–19].

Metachronal beating of cilia under the influence of Casson fluid [20] and magnetic field is considered. The model for cilia literature is modelled for the first time. The governing coupled equations are constructed under long wavelength and low Reynold's number approximation. Exact solutions are evaluated for stream function and pressure gradient. The important results in this study are the variation of the Hartmann number M and Casson fluid parameter ζ.

2. Flow equations

Let us consider the flow of an incompressible Casson fluid through a symmetric human body. We chose the Cartesian coordinates (Y, X), where X-axis lies along the center of the body and Y is transverse to it. Flow is generated due to the metachronal wave which is produced due to collective beating of the cilia with constant speed c along the walls of the body whose inner surface

* Corresponding author. Tel.: +92 5190642182; fax: +92 512275341.
E-mail address: noreensher@yahoo.com (N.S. Akbar).

http://dx.doi.org/10.1016/j.jmmm.2014.11.056
0304-8853/

Nomenclature

ϵ	ratio w.r.t cilia length
P	pressure
x	variable along the body
$\bar{\tau}$	extra stress tensor
λ	wavelength
β	wave number
y	variable horizontal to the body
a	mean radius of the tube
u,v	velocities
x	variable normal to the channel
μ	fluid viscosity
c	wave speed
ζ	Casson fluid parameter
α	measure of the eccentricity

is ciliated (Fig. 1). The geometry of the wall surface is given by

$$\bar{Y} = f(\bar{X}, t) = \pm\left[a + a\epsilon\cos\left(\frac{2\pi}{\lambda}\left(\bar{X} - c_1 t\right)\right)\right] = \pm L = \pm H, \tag{1}$$

Sleigh [1] observed that cilia tips move in elliptical paths, therefore, the vertical position of the cilia tips can be written as

$$\bar{X} = g(\bar{X}, t) = X_0 + a\epsilon\alpha\sin\left(\frac{2\pi}{\lambda}\left(\bar{X} - c_1 t\right)\right), \tag{2}$$

The horizontal and vertical velocities of the cilia are given as [11]

$$U_0 = \frac{-\left(\frac{2\pi}{\lambda}\right)a\epsilon\alpha c_1\cos\left(\frac{2\pi}{\lambda}\left(\bar{X} - c_1 t\right)\right)}{1 - \left(\frac{2\pi}{\lambda}\right)a\epsilon\alpha c_1\cos\left(\frac{2\pi}{\lambda}\left(\bar{X} - c_1 t\right)\right)}, \tag{3}$$

$$V_0 = \frac{-\left(\frac{2\pi}{\lambda}\right)a\epsilon\alpha c_1\sin\left(\frac{2\pi}{\lambda}\left(\bar{X} - c_1 t\right)\right)}{1 - \left(\frac{2\pi}{\lambda}\right)a\epsilon\alpha c_1\sin\left(\frac{2\pi}{\lambda}\left(\bar{X} - c_1 t\right)\right)}. \tag{4}$$

The expression for fixed and wave frames is related by the following relations:

$$\bar{x} = \bar{X} - ct, \quad \bar{y} = \bar{Y}, \quad \bar{u} = \bar{U} - c, \quad \bar{v} = \bar{V}, \quad p(\bar{x}) = P(\bar{X}, t). \tag{5}$$

The constitutive equation for Casson fluids [20] is defined as follows:

$$\tau^{1/n} = \tau_0^{1/n} + \mu\dot{\gamma}^{1/n} \tag{6a}$$

$$\tau_{ij} = \left(\mu_B + \sqrt{2\pi_c}/p_y\right)2e_{ij} \tag{6b}$$

where p_y is the yield stress and $\pi = e_{ij}$, e_{ij} is the (i, j) component of deformation rate, μ_β is the plastic viscosity of the fluid. For ordinary fluid $\left(p_y = 0\right)$, we have

$$e_{ij} = \frac{1}{2}\left(\frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial yi}\right). \tag{6c}$$

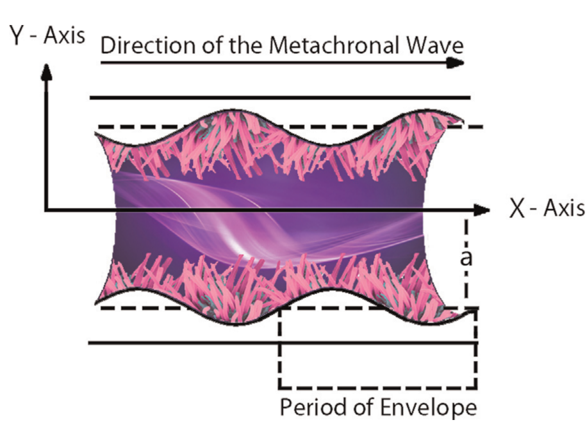

Fig. 1. Geometry of the problem.

we introduce the following non-dimensional quantities:

$$x = \frac{2\pi\bar{x}}{\lambda}, \quad y = \frac{\bar{y}}{a}, \quad u = \frac{\bar{u}}{c}, \quad v = \frac{\bar{v}}{c}, \quad t = \frac{2\pi\bar{t}}{\lambda}, \quad \beta = \frac{2\pi a}{\lambda},$$
$$P = \frac{2\pi a^2\bar{P}}{\mu c\lambda}, \ h = \frac{\bar{h}}{a}, \quad \mathrm{Re} = \frac{\rho ca}{\mu}, \quad S = \frac{\bar{S}a}{\mu c}, \quad \zeta = \mu_\beta\sqrt{2\pi_c}/p_y. \tag{7}$$

where ζ denotes the Casson fluid parameter.

In view of Eqs. (2)–(7) under the long wavelength and low Reynolds number assumption we have the following equations:

$$\left(1 + \frac{1}{\zeta}\right)\frac{\partial^2 u}{\partial y^2} - M^2(u + 1) = \frac{dp}{dx}, \tag{8}$$

$$\frac{\partial u}{\partial y} = 0, \quad \text{at } y = 0, \tag{9a}$$

$$u = -1 - \frac{2\pi\epsilon\alpha\beta\cos(2\pi x)}{1 - 2\pi\epsilon\alpha\beta\cos(2\pi x)}, \ \text{at } y = h = 1 + \epsilon\cos(2\pi x), \tag{9b}$$

Flow rate in dimensionless form can be written as follows:

$$Q = F + 1. \tag{10}$$

3. Solution profiles

Exact solutions for velocity and pressure gradient can be written as

$$u(x, y) = -1 + \frac{dp}{dx}\frac{1}{M^2} + \frac{1}{2}\left(-\frac{2\pi\epsilon\alpha\beta\cos(2\pi x)}{1 - 2\pi\epsilon\alpha\beta\cos(2\pi x)} - \frac{dp}{dx}\frac{1}{M^2}\right)$$
$$\times \operatorname{sech}\left(\frac{\sqrt{\zeta}hM}{\sqrt{\zeta + 1}}\right)\left(\sinh\left(\frac{\sqrt{\zeta}My}{\sqrt{\zeta + 1}}\right) + \cosh\left(\frac{\sqrt{\zeta}My}{\sqrt{\zeta + 1}}\right)\right)$$
$$- \frac{\left(AM^2 + \frac{dp}{dx}\right)\left(\sinh\frac{\sqrt{\zeta}hM}{\sqrt{\zeta + 1}}(M - y) + \cosh\frac{\sqrt{\zeta}hM}{\sqrt{\zeta + 1}}(M - y)\right)}{M^2\left(\sinh\left(\frac{2\sqrt{\zeta}hM}{\sqrt{\zeta + 1}}\right) + \cosh\left(\frac{2\sqrt{\zeta}hM}{\sqrt{\zeta + 1}}\right) + 1\right)}. \tag{11}$$