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Current Perspectives

Extension of Bilbro–McMillan charge density wave-superconductivity coexistence relation to quantum régimes: Application to superconducting domes around quantum critical points

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ABSTRACT

Quantum critical points (QCP) accompanied by superconductivity are ubiquitous in condensed matter physics. In general, the transition temperature T_0 of an ordered state, e.g. antiferromagnetic, goes to zero under the influence of an external parameter, e.g. pressure. Superconductivity appears before the disappearance of the ordered state, but reaches its maximum T_c when $T_0 = 0$. Presently, the implications of the QCP's on superconductivity are a subject of debate. It is proposed here that both transition temperatures satisfy the relation $\tilde{T}_0^{\,2} + \tilde{T}_c^{\,2} = 1$, where the tilde indicates normalization to the maximum values. Inspired from the basic postulate of SO(N) theories of superconductors, it is proposed as an extension from Bilbro–McMillan relation.

The study of phase transitions is central to the understanding of the physical properties of condensed matter [1]. Standard phase transitions, at a transition temperature $T_c \neq 0$, can be treated classically, even those between different quantum states. Quantum phase transitions, where the transition between two different phases takes place at $T_c = 0$, are qualitatively different, as their critical fluctuations must be treated quantum mechanically [2,3]. Great progress has been made in their understanding, e.g. in the case of magnetic quantum phase transitions [4–6]. In a large number of materials including pnictides, heavy fermions, charge or spin density wave materials, the phase transition at $T_c = 0$ as a function of a certain parameter δ (pressure, doping, etc.) from an ordered, generally bad conductor, state to a conducting disordered phase, is surrounded in the $T-\delta$ plane by a superconducting region (see Fig. 1). The central point is that there is a region where the ordered and the superconducting state (SC) coexist. For charge density waves (CDW) this coexistence has been studied by several groups [7–9]. In particular, Bilbro and McMillan (BM) [8] studied the interaction of CDW and SC in A15 materials within mean field. Here, both states compete for developing their respective gaps in the same Fermi surface. They obtained the following relation between T_c, the superconducting transition temperature, and T_0, the CDW transition temperature,

$$T_0^{1-n_0}(P)T_c^{\,n_0}(P) = T_{c_{Max}} \tag{1}$$

where n_0 is the fraction of the Fermi surface under the CDW gap and $T_{c_{Max}}$ the superconducting transition with no CDW.

Different works have shown that under pressure there are clearly two regimes for the evolution of T_0 on its way to the QCP. First, at low pressures, a classical BCS one and then a quantum fluctuations (QF) one near the QCP. For example, both the one dimensional transition metal trichalcogenide $NbSe_3$, a well-studied charge density wave (CDW) compound [10–12], and Cr metal [13] display, at low pressures, a decrease of T_0 following an exponentialwith pressure BCS tuned variation, a consequence of the dependence of T_0 on the coupling parameter λ, i.e. $T_0 \sim e^{-1/\lambda}$. A quantum region appears below a crossover value of T_0 where QF takes control through a power law behavior with an exponent $\Psi = ½$ (see the behavior of $Ag_{0.33}V_2O_5$ in Fig. 1).

It is clear that Eq. (1) can only be applied in the BCS régime and is not valid in the QF region. However, at least for CDW and SDW materials, a direct relation between T_0 and T_C describing that both states are the result of the same Fermi surface and electron–phonon interaction is expected and needed. In order to have a hint towards it, an analysis of how the BM relation works experimentally is clarifying.

In A-15 compounds, $n_0 \sim$ some 10%'s and changes only a few percent in the measured pressure range [8]. Considering that n_0 is approximately constant in the pressure range, expression (1) becomes,

$$(1-n_0)\ln T_c(P) + n_0 \ln T_0(P) \approx \ln T_{c_{Max}} \tag{2}$$

We see from Fig. 2 where the variation with pressure of both transition temperatures for the two most studied A-15 compounds, V_3Si and Nb_3Sn is plotted as logarithms, that expression (2) is verified,

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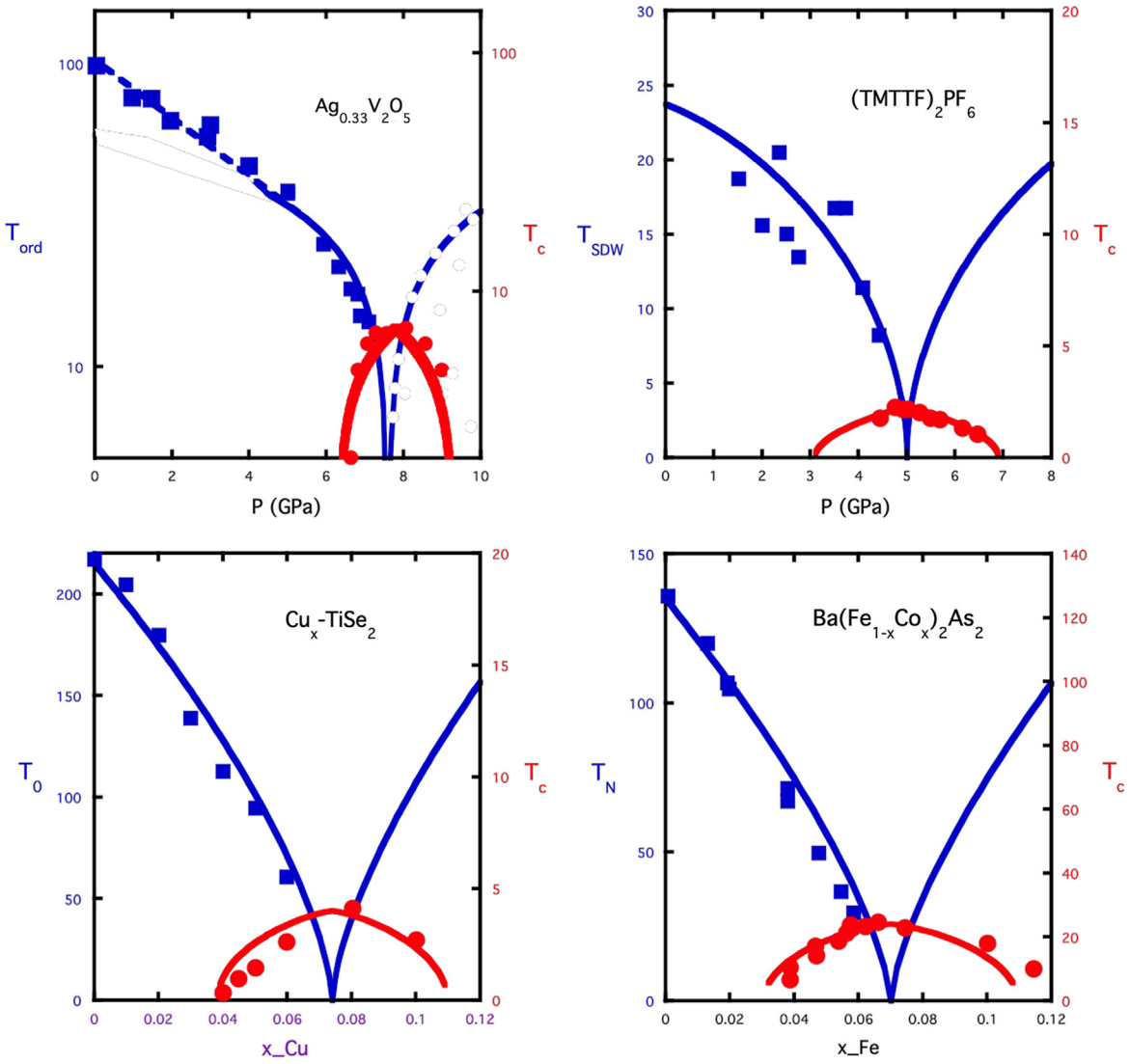

Fig. 1. Examples of materials showing a QCP surrounded by a superconducting dome. The compound ($Ag_{0.33}V_2O_5$) [22] follows at low pressures an exponential with pressure BCS tuned variation up to ~5 GPa (blue dashed right line). Above this pressure, the compound crossovers to a quantum fluctuations regime, following the power law of the type $T_0 \propto [(\delta_c - P)/\delta_c]^{1/2}$ (blue solid line). The other three compounds $(TMTTF)_2PF_6$ [23], Cu_xTiSe_2 [24] and $Ba(Fe_{1-x}Co_x)_2As_2$ [25] show power laws at all pressures. Also shown (red circles) is the superconducting dome. The red line is obtained using formula (4), see text and Fig. 3.

n_0 being constant within experimental error, and 0.26 for V_3Si and 0.14 for Nb_3Sn; the obtained $T_{c_{Max}}$ are 17.7 K and 20.1 K for V_3Si and Nb_3Sn, respectively. Experimentally we have $\ln T_x(P) = \ln T_x(0) + \alpha_x P$, α_x being a constant and x either c or 0. The logarithmic variation with pressure of both transition temperatures is the result of the exponentially tuned BCS expression. In other words, at all the shown pressures, both transitions without any doubt obey BCS statistics.

However, as described above, the CDW/SDW transition deviates from the exponentially tuned BCS behavior towards a power law dependence of the type $T_0 \propto [(\delta_c - \delta)/\delta_c]^{1/2}$ near to the QCP. It is apparent that the transition temperature does not follow BCS statistics, but another possibly due to the QF near the QCP. QF accelerates the rhythm of decrease of T_0, destabilizing the CDW/SDW and, at the same time, inducing a faster transfer of carriers from the CDW/SDW gap to the SC gap. Eq. (1) is thus no longer valid. Phenomenologically, expression (1) adds the variations of the $T_x^{'S}$ using the functional that is linear with pressure, which in the BCS régime is the logarithm of the transition temperature. The same approach may be taken for the power law régime. Thus, instead of the logarithms, as now $[T_0/T_0(0)]^2 = 1 - P/P_c$, the squares of the transition temperatures should be added and equated to a constant. It is thus expected that the equivalent of BM for the power law region would be a summation of squares of the type

$$aT_0^2 + bT_c^2 = \kappa \tag{3}$$

To advance further in obtaining a BM formula for the quantum régime, it is useful to take into account the SO(5) formulation. In the paper [14] on SO(5), it was argued that the simplest way to construct an unified theory of antiferromagnetism and superconductivity for cuprates was to define a five dimensional spinor composed of the three components of the magnetization and the real and imaginary parts of the d-wave superconducting gap. In this way, it would be possible to recover all the relevant physics of the problem of high temperatures superconducting cuprates [15]. More complicated problems would require higher dimensions spinors to obtain all the physics, i.e. the complete phase diagram with all the possible ground states, e.g. SO(8) for a two chain model of correlated fermions [16]. However, experimentalists can be less ambitious, and just try to describe phenomenologically with the most compact expression a restricted region of the phase diagram, e.g. the one near QCP. Basically, there are two coexisting phases. The ordered phase should rigorously have several components. However, systems with presumed different number of components, show the same behavior near the QCP. SO(5) and SO(4) have