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Negative electric susceptibility and magnetism from translational invariance and rotational invariance

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ABSTRACT

In this work we investigate magnetic effects in terms of the translational and rotational invariances of magnetisation. Whilst Landau-type diamagnetism originates from translational invariance, a new diamagnetism could result from rotational invariance. Translational invariance results in only conventional Landau-type diamagnetism, whereas rotational invariance can induce a paramagnetic susceptibility for localised electrons and also a new kind of diamagnetism that is specific to conducting electrons. In solids, the moving electron shows a paramagnetic susceptibility but the surrounding screening of electrons may produce a new diamagnetic response by Lenz's law, resulting in a total susceptibility that tends to zero. For electricity, similar behaviours are obtained. We also derive the DC-type negative electric susceptibility via two methods in analogy with Landau diamagnetism.

Magnetism in modern solid state physics remains the focus of many theoretical and experimental studies [1]. Magnetic materials are conventionally classified as paramagnetic, ferromagnetic, antiferromagnetic or diamagnetic [2]; also of importance are the theoretical models such as the phonon-assisted mechanism proposed by Kim [3,4], and the spin fluctuation scheme [5] for metallic ferromagnetism. Many scientists have questioned whether or not a Pauli-type diamagnetism can occur, and also whether a Landau-type paramagnetism may be possible [3,4]. In ferromagnetism, ferromagnetic insulators are explained by localised Heisenberg spins according to Curie–Weiss law and this theory is well accepted with some consensus among the magnetic community, and can be applied to metallic ferromagnetism [6] using localised block spins. In recent years, the Berry phases [7] related to rotational invariance have been considered important. In optics and particle physics, rotational anomalies of lights or particles are categorised as chiral phenomena. For an external magnetic field, the field is expelled from within the superconducting material, and external materials with nonzero magnetic fields placed near the superconducting material levitate away from the superconductor. This levitation results from a negative magnetic response known as diamagnetism. It is possible for some materials to have negative electric response states known as **diaelectric** states in analogy with the diamagnetic states found in magnetism. Outside the **diaelectric** material the charges may levitate. It is generally thought that this state, termed supermagnetic, has already been realised, having been confirmed by indirect evidence. However, no direct detection of supermagnetism has been achieved to date; furthermore, the supermagnetism thought to have been found in conventional experiments may be regarded not as a **diaelectric** state at all, but merely as normal electric phases due to the polarity of the electric charge. The candidate materials suitable for realising **diaelectric** states are likely to be a kind of unknown meta-material of DC-type, i.e., $\varepsilon'(\omega = 0) < 0$, $\mu(\omega = 0) < 0$, where μ is the magnetic permeability. The other strong candidate materials that could show **diaelectric** behaviours require careful cheques to be made using V and Nb as examples.

We herein intend to **verify** conventional diamagnetism and paramagnetism, and investigate new magnetisms using rigorous considerations of invariance. We also revisit magnetism from the perspective of the rotational and translational invariances of magnetisation. We herein also derive negative electric susceptibility using two methods in an analogy with Landau diamagnetism.

Let us first consider translational invariance.

The magnetisation M obeys

$$M(\vec{r} + \vec{T}) = M(\vec{r}) \tag{1}$$

where $\vec{T}$ is the translational vector, and $\vec{r}$ is the positional vector.

We also have

$$\vec{k} \cdot \vec{T} = 0 \Rightarrow \vec{k} \perp \vec{T} \tag{2}$$

where $\vec{k}$ is the wavevector.

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Landau-type diamagnetism [2,4] stems from this condition in the sense that the orbital surface is as important as the Landau's surface.

In the case of rotational invariance, we have

$$\vec{\nabla} \times \vec{M}`(\vec{r}) = 0 \tag{3}$$

where $\vec{M}`$ is the magnetic moment.

Using Stokes' theorem, we obtain

$$\int_A \vec{\nabla} \times \vec{M}`\cdot d\vec{S} = \oint \vec{M}`\cdot d\vec{l} \tag{4}$$

and the susceptibility is given by

$$\tilde{\chi} = \frac{\partial M}{\partial H} = M \tan \theta \frac{\partial \theta}{\partial H}, \tag{5}$$

where θ is the phase angle between the positional vector and magnetic field, and H is the magnetic field.

Itinerant electrons are governed by the Lorentz force as follows.

$$\vec{F} = q\vec{v} \times \vec{H}|\vec{F}| = qvH \sin \theta \frac{\partial |\vec{F}|}{\partial H} = |qvH| \cos \theta \frac{\partial \theta}{\partial H} + |qv| \sin \theta \tag{6}$$

where q is the charge and v is the electron velocity.

The screening of a moving electron is induced by surrounding electrons that oppose the Lorentz forces according to Lenz's law to give $(\partial\theta/\partial H) < 0$. In other words, the moving electron shows a paramagnetic susceptibility $(\chi_1 > 0)$ but the induced screening of electrons may produce a diamagnetic response by Lenz's law $(\chi_2 < 0)$, resulting in a total susceptibility that tends to zero $(\chi_1 + \chi_2 \to 0)$.

By analogy with image-charge methods, the screening electrons are governed by the imaginary component of the target electron, on the basis of Maxwell's equations.

We now derive a corrected version of the imaginary part of Pauli paramagnetism for non-interacting electrons.

In a magnetic field H applied in the positive z direction, the electron number densities are given by

$$n_+ = \int d\varepsilon N(\varepsilon - \mu_B H) f(\varepsilon) \to \int d\varepsilon N(\varepsilon - \mu_B H) f_{BE}(\varepsilon) \tag{7}$$

where $n_+(n_-)$ represents the number density for up (down) spins, $N(\varepsilon)$ is the density of states,$f(\varepsilon)$ is the Fermi–Dirac distribution, μ_B is the Bohr magneton, and f_{BE} is the Bose–Einstein distribution corresponding to the imaginary part of f, and the imaginary argument of the Fermi–Dirac distribution is given by

$$\begin{aligned} f(\varepsilon) &= \frac{1}{1 + \exp(\varepsilon - \mu/k_B T)} \\ &= \frac{\exp(i(\varepsilon - \mu/2k_B T)i)}{\exp(i(\varepsilon - \mu/2k_B T)i) + \exp(-i(\varepsilon - \mu/2k_B T)i)} \\ &\to \frac{-\exp\{-i((\pi/2) - (\varepsilon - \mu/2k_B T)i)\}}{\exp\{-i((\pi/2) - (\varepsilon - \mu/2k_B T)i)\} + \exp\{i((\pi/2) - (\varepsilon - \mu/2k_B T)i)\}} \\ &= \frac{1}{\exp(\varepsilon - \mu/k_B T) - 1} = f_{BE}(\varepsilon) \end{aligned} \tag{8}$$

where the imaginary part is $\pm f(\pm((\pi/2) - \theta))$for the real part of $f(\theta)$ with phase angle θ.

The resultant imaginary magnetic susceptibility is then given by

$$\chi`` = \frac{-\mu_B(n_+ - n_-)}{H} \approx 2\mu_B^2 (N`(\varepsilon_F)\varepsilon_F), \tag{9}$$

where ε_F is the Fermi energy and $N'(\varepsilon_F)$ is a 1^{st} derivative about ε, and is positive in most materials.

In contrast to Landau diamagnetism, a new kind of diamagnetism known as **Pauli diamagnetism**, for V, Nb with $N'(\varepsilon_F) < 0$ may be expressed as

$$\chi_{Pauli}^{Diamagnetism} \equiv -2\mu_B{}^2|N'(\varepsilon_F)|\varepsilon_F. \tag{10}$$

We propose to make corrections to the above **Pauli diamagnetism** by imposing a relationship between the thermal activation (temperature) and the magnetic field-induced excitation (magnetic field). In other words, we consider a relationship between temperature T and magnetic field H in the system of Fermi electrons.

The average number density of an electron in the absence of H is given by

$$\begin{aligned} \int_0^\infty f(\epsilon)d\epsilon &= \int_0^\infty \frac{d\epsilon}{1 + \exp(\epsilon - \epsilon_F/k_B T)} \\ &= \int_0^\infty \frac{d\epsilon}{1 + \exp(\epsilon \pm \mu_B H - \epsilon_F/k_B T_{eff})} \\ &\equiv k_B T \ln[1 + \exp(\epsilon_F/k_B T)] \\ &= k_B T_{eff} \ln[1 + \exp(-(\pm \mu_B H - \epsilon_F)/k_B T_{eff})] \end{aligned} \tag{11}$$

where $f(\varepsilon)$ is the Fermi–Dirac distribution, $N(\varepsilon)$ is the density of states, β is a positive constant parameter and ε_F is the Fermi energy. The total energy of an electron in the presence of H along the z-axis can be expressed in the form:

$$\frac{1}{\beta} k_B T^{eff} \equiv \frac{1}{\beta} k_B T \pm \mu_B H \tag{12}$$

where T^{eff} defines the effective temperature. We derive a corrected version of Pauli diamagnetism that incorporates the temperature dependence into the magnetic field dependence. For a magnetic field H applied in the positive z direction, from Eq. we obtain

$$\begin{aligned} n_+ &= \int d\varepsilon N(\varepsilon - \mu_B H) f_{BE}(\varepsilon)|_{k_B T^{eff} = k_B T - \beta\mu_B H} \\ n_- &= \int d\varepsilon N(\varepsilon + \mu_B H) f_{BE}(\varepsilon)|_{k_B T^{eff} = k_B T + \beta\mu_B H} \end{aligned} \tag{13}$$

where n_+ (n_-) represents the number density for up (down) spins, $N(\varepsilon)$ is the density of states,$f_{BE}(\varepsilon)$ is the Bose–Einstein distribution, and μ_B is the Bohr magneton. The Bose–Einstein distribution is given by

$$\begin{aligned} & f_{BE}(\varepsilon)|_{k_B T^{eff} = k_B T \pm \beta\mu_B H} \\ &= \frac{1}{-1 + \exp(\varepsilon - \varepsilon_F/k_B T \pm \beta\mu_B H)} \\ &\approx f_{BE}(\varepsilon) \mp \beta(\varepsilon - \varepsilon_F) f'_{BE}(\varepsilon) \frac{\mu_B H}{k_B T} \end{aligned} \tag{14}$$

in which we consider the case $\mu_B H << k_B T$, and ε_F is the Fermi energy.

The resultant magnetic susceptibility is then given by

$$\chi = \frac{-\mu_B(n_+ - n_-)}{H} = 2\mu_B^2 N'(\varepsilon_F)\varepsilon_F + 4\mu_B^2 N(\varepsilon_F)\frac{\beta\varepsilon_F}{k_B T} + \cdots \tag{15}$$

We use the electric vortex concept, as introduced for quark confinement [8]. The effective charge for an electron attached to an electric flux is termed a composite charge as shown in Fig.1 such that

$$\begin{aligned} -e(EA \pm pE_0 A_0) &= e_* EA \\ E_0 A_0 &= \frac{1}{l} EA \end{aligned} \tag{16}$$

where $e_*$is the effective charge and $E(E_0)$, $A(A_0)$are the electric field and surface in part of the bulk (in an electric cylindrical vortex), ε is the electric permittivity inside the materials, p is the number of electric field lines in an electric vortex attached to an