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Magnetic properties of a long, thin-walled ferromagnetic nanotube

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ABSTRACT

We consider magnetic properties of a long, thin-walled ferromagnetic nanotube. We assume that the tube consists of isotropic homogeneous magnet whose spins interact via the exchange energy, the dipole–dipole interaction energy, and also interact with an external field via Zeeman energy. Possible stable states are the parallel state with the magnetization along the axis of the tube, and the vortex state with the magnetization along azimuthal direction. For a given material, which of them has lower energy depends on the value $\gamma = R^2 d/(L\lambda_x^2)$, where R is the radius of the tube, d is its thickness, L is its length and λ_x is an intrinsic scale of length characterizing the ratio of exchange and dipolar interaction. At $\gamma < 1$, the parallel state wins, otherwise the vortex state is stable. A domain wall in the middle of the tube is always energy unfavorable, but it can exist as a metastable structure. Near the ends of a tube magnetized parallel to the axis a half-domain structure transforming gradually the parallel magnetization to a vortex just at the edge of the tube is energy favorable. We also consider the equilibrium magnetization textures in an external magnetic field either parallel or perpendicular to the tube. Finally, magnetic field produced by a nanotube and an array of tubes is analyzed.

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1. Introduction

Magnetic nanomaterials play an important role in applications as elements of memory and magnetic sensors and switches as it was demonstrated by Nobel prize 2007 to Fert and Grünberg for their invention of antiferromagnetic spin valve. The tasks of further miniaturization of magnetic devices and creation of configurations providing a controllable magnetic field are extremely important for nanophysics and technology. Experimenters and technologists have already created nanomagnets in different shapes – disks [1], rings [2], wires [3], etc. Among these new nanomaterials the nanotubes, as compared to solid wires, have inner voids that reduce the density of materials and make them easier to float in solutions, a desirable property in biotechnology [4]. The inner hollow itself can be used for capturing large biomolecules [5]. Besides, as magnetic materials, they are free of vortex cores, which make the vortex state more stable than that of nanowires. This makes nanotubes more suitable as candidates for elements of memory for computers and as a tool for creation of superconductors with high critical fields. Several methods have been used to synthesize nanotubes: electrodeposition [6,7], atomic layer deposition [8], hydrogen reduction [9]. Ferromagnetic materials used for formation of nanotubes include Ni, [8], Co [6,8], FePt [9], and Fe_3O_4 [9].

Together with the experimental progress, theoretical calculations and numerical simulations for nanotubes were performed extensively, dealing with the stable states [4,10], switching behavior [11], hysteresis loop [4,12] and properties of domain walls (both static [11] or dynamic [13]). In Ref. [10] the authors calculated numerically and partly analytically energy of the parallel state and the vortex state as a function of the dimensions of the tube and material constants. Phase diagrams were drawn in terms of linear dimensions of the tube. In Ref. [4] the authors have shown that the parallel magnetization turns into a vortex-like one at the edge of the tube.

The purpose of our work is to give an analytical description of the magnetic tube (MT) properties employing small parameters characterizing their geometry: the ratios d/R and R/L. In the experimentally realized MT the first ratio was in the range of 10^{-3} and the second one varied between 10^{-2} and 10^{-1}. The analytical approach allows us to construct the complete phase diagram of the MT in the space of geometric parameters and external magnetic field. We establish analytical criteria for the appearance and disappearance of different topological magnetic configurations, topological defects and field-induced magnetic textures. We also calculate the magnetic field produced by the tubes.

2. The model

We take into account the magnetic interactions of two kinds: the exchange interaction and the dipolar interaction. The total

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energy of a MT is

$$E = E_{exch} + E_{dip}$$
$$= -J \sum_{\langle \mathbf{x},\mathbf{x}' \rangle} \mathbf{S}_{\mathbf{x}} \cdot \mathbf{S}_{\mathbf{x}'} + \frac{\mu_0}{4\pi}(g\mu_B)^2 \sum_{\mathbf{x},\mathbf{x}',\mathbf{x}\neq\mathbf{x}'} \frac{\mathbf{S}_{\mathbf{x}} \cdot \mathbf{S}_{\mathbf{x}'} - 3(\mathbf{S}_{\mathbf{x}} \cdot \hat{\mathbf{r}})(\mathbf{S}_{\mathbf{x}'} \cdot \hat{\mathbf{r}})}{r^3}. \quad (1)$$

Here $\mathbf{S}_{\mathbf{x}}$ is the spin vector at position $\mathbf{x}$, J is the exchange constant, $\hat{\mathbf{r}}$ is the unit vector from position $\mathbf{x}$ to $\mathbf{x}'$, $\langle \mathbf{x},\mathbf{x}' \rangle$ means summation over all nearest pairs, and $r = |\mathbf{r}| = |\mathbf{x}-\mathbf{x}'|$ [14]. Further we use the International System of units. Another often considered contribution to the total energy, the crystal anisotropy, is not included here. This is appropriate when the material is a polycrystal with large number of randomly orientated grains like permalloy. Experimenters [6] indicate that the size of a single-crystal grain in their nanotubes is about 1 nm. We do not know how strong is the exchange interaction between the grains. In our calculations we assume that it is the same as in the bulk single crystal.

We adopt an approximation of classical continuous field $\mathbf{m}(\mathbf{x})$ for the magnetic order parameter with the constraint $\mathbf{m}^2(\mathbf{x}) = 1$. The magnetization at the point $\mathbf{x}$ of the space is equal to $M_0\mathbf{m}(\mathbf{x})$. The saturation magnetization M_0 is assumed to be dependent on temperature, but independent of the point $\mathbf{x}$ of the space. In this approximation [15] the exchange energy reads as follows:

$$E_{exch} = A \int d^3x (\nabla \mathbf{m}(\mathbf{x}))^2, \quad (2)$$

where $A = (1/6)Jns^2Za^2$, n is the number of magnetic atoms per unit volume, s is the magnitude of their spin, $(\nabla \mathbf{m}(\mathbf{x}))^2 = (\nabla m_x(\mathbf{x}))^2 + (\nabla m_y(\mathbf{x}))^2 + (\nabla m_z(\mathbf{x}))^2$, Z is the coordination number, and a is the distance between two nearest atoms.

There are several equivalent expressions for the dipolar energy:

$$E_{dip} = \frac{1}{2}\frac{\mu_0}{4\pi} M_0^2 \int d^3x\, d^3x' \frac{\mathbf{m}(\mathbf{x}) \cdot \mathbf{m}(\mathbf{x}') - 3(\mathbf{m}(\mathbf{x}) \cdot \hat{\mathbf{r}})(\mathbf{m}(\mathbf{x}') \cdot \hat{\mathbf{r}})}{r^3}, \quad (3a)$$

$$E_{dip} = \frac{1}{2}\frac{\mu_0}{4\pi} M_0^2 \int d^3x\, d^3x' (\mathbf{m}(\mathbf{x}) \cdot \nabla_{\mathbf{x}})(\mathbf{m}(\mathbf{x}') \cdot \nabla_{\mathbf{x}'}) \frac{1}{r}, \quad (3b)$$

$$E_{dip} = \frac{1}{2}\frac{\mu_0}{4\pi} M_0^2 \left[\int dA\, dA' \frac{\sigma_M(\mathbf{x})\sigma_M(\mathbf{x}')}{r} + 2 \int dA\, d^3x' \frac{\sigma_M(\mathbf{x})\rho_M(\mathbf{x}')}{r} + \int d^3x\, d^3x' \frac{\rho_M(\mathbf{x})\rho_M(\mathbf{x}')}{r} \right]. \quad (3c)$$

The integration denoted by $\int dA$, $\int dA'$ proceeds over the surfaces of the magnet, the integration denoted as $\int d^3x$ goes over its volume. The value $\sigma_M(\mathbf{x}) = \mathbf{m}(\mathbf{x}) \cdot \mathbf{n}$ is the "surface magnetic charge density" and $\rho_M(\mathbf{x}) = -\nabla_x \cdot \mathbf{m}(\mathbf{x})$ is the "volume magnetic charge density". Eq. (3c) is a form analogous to the energy of electric charges interacting via Coulomb forces. This analogy allows us to use results well-known in electrostatics. An important consequence of this analogy is that the dipolar energy is non-negative, since the electrostatic energy is equal to the integral of the square of the electric field. Eq. (3c) provides a clear electrostatic visualization of the dipolar interaction. Eq. (3b) may occur more convenient for specific calculations. A system of magnetic charges is always neutral.

3. Stable states

We consider a cylindrical tube located between $z = -L/2$ and $z = L/2$, as shown in Fig. 1, with the radius R, thickness d, and length L. We assume $d \ll R \ll L$. This research was initially stimulated by a new material fabricated experimentally by Dr. Wenhao Wu and his group at Texas A&M University: an array of nickel nanotubes in alumina with dimensions approximately $R = 150$ nm, $d = 30$ nm and $L = 60$ μm. For these nanotubes the condition $R \ll L$ is well satisfied, while the condition $d \ll R$ is relatively not so well satisfied. Besides, $R^2 \ll dL$ is also well satisfied. In earlier experiments [1–9] all three strong inequalities were satisfied.

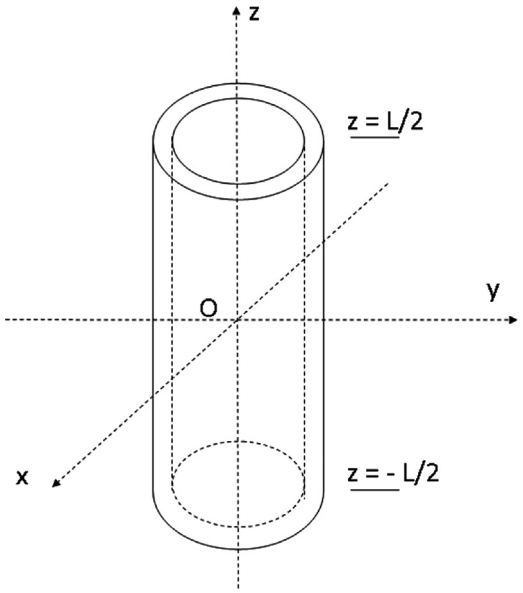

Fig. 1. Geometry of a tube, with two edges located at $z = L/2$ and $z = -L/2$.

Natural candidates to the state with the lowest energy are the most symmetric magnetic configurations: the parallel state: $\mathbf{m}(\mathbf{x}) = \hat{z}$, the vortex state: $\mathbf{m}(\mathbf{x}) = \hat{\phi}$, and the radial state: $\mathbf{m}(\mathbf{x}) = \hat{\rho}$. We denote azimuthal angle as ϕ; the symbol $\hat{\phi}$ denotes the unit vector in azimuthal direction. Each of these states is two-fold degenerate due to time reversal invariance. We also consider two other less symmetric states: the transverse state: $\mathbf{m}(\mathbf{x}) = \hat{x}$, and the so-called onion state. According to Ref. [18], the onion state (see Fig. 2) becomes stable in ferromagnetic rings in some range of parameters. Here we consider its analogue for a tube. These two kinds of states occur to be stable magnetic configuration in the transverse magnetic field.

In the parallel state, $\nabla \mathbf{m}(\mathbf{x}) = 0$, $\rho_M(\mathbf{x}) = 0$, $\sigma_M(\mathbf{x}) = \pm 1$ for $z = \pm L/2$, and $\sigma_M(\mathbf{x}) = 0$ elsewhere. The exchange energy is zero. The dipolar energy consists of three parts: the self-energies of the two edges and the energy of interaction between. Since $R \ll L$, the latter term is much smaller than the former two and further we neglect it. The distance r between two points with cylindrical coordinates (ρ, ϕ, z) and (ρ', ϕ', z') belonging to a MT satisfying the inequality $d \ll R$ reads as follows: $r \approx \sqrt{(\rho-\rho')^2 + 2R^2[1-\cos(\phi-\phi')] + (z-z')^2}$. Each self-energy term after integration over ϕ and ϕ' is reduced to an integral of the complete elliptic integral of the first kind $K(k)$ where $k = \sqrt{4\rho\rho'/(\rho+\rho')^2}$. The condition $d \ll R$ allows us to use the approximation $K(k) \approx 2 \log 2 - \frac{1}{2}\log(1-k^2)$ since k is close to 1. In this approximation the integration over ρ and ρ' is straightforward leading to the result for the self-energy of each edge: $E(\text{edge}) = \frac{1}{2}\mu_0 M_0^2 R d^2 (\log 8R/d + \frac{3}{2})$. Thus, in the limit of long thin MT the total energy of the parallel state is $E_P = \mu_0 M_0^2 R d^2 (\log 8R/d + \frac{3}{2})$. It does not depend on the tube length L.

Different magnetic configurations and results of similar calculations of exchange and dipolar energy for them are summarized in Table 1.

While other configurations are trivial, some comments on the onion configuration are necessary. We seek for a variational distribution of magnetization that satisfies following requirements: magnetization must be parallel to magnetic field (in the direction $\hat{x}$) at $\phi = 0, \pm\pi/2, \pi$ and it does not depend on the coordinate ρ in a narrow ring. A simplest vector field satisfying these requirements has a form $\mathbf{m}(\phi) = \hat{x}\cos\theta(\phi) + \hat{y}\sin\theta(\phi)$