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Consequence of nanofluid on peristaltic transport of a hyperbolic tangent fluid model in the occurrence of apt (tending) magnetic field

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ABSTRACT

In the current study, sway of nanofluid on peristaltic transport of a hyperbolic tangent fluid model in the incidence of tending magnetic field has been argued. The governing equations of a nanofluid are first modeled and then simplified under lubrication approach. The coupled nonlinear equations of temperature and nano particle volume fraction are solved analytically using a homotopy perturbation technique. The analytical solution of the stream function and pressure gradient are carried out using perturbation technique. The graphical results of the problem under discussion are also being brought under consideration to see the behavior of various physical parameters.

1. Introduction

Peristaltic transports are a vigorous research area because of their spacious range applications in physiology and industry. Such flows occurs in urine transport from kidney to bladder, swallowing food through the esophagus, mixing of food and chyme movement in the intestine, circulation of blood in small blood vessels and blood pumps in heart lung machines. To recognize peristaltic action numerous theoretical and experimental studies have been accomplished [1–10].

In recent years, the study of MHD flow problems has achieved significant interest because of its wide-ranging engineering and medical applications [11–14]. An effect of inclined magnetic field on magneto fluid flow through porous medium between two inclined wavy porous plates was explored in [15]. Recently, Nadeem and Akram [16] have discussed the inclined magnetic field in viscous peristaltic phenomena in presence of heat and mass transfer, where an exact solution of reduced equations has been carried out. The study of nano fluids is another important area which has recently attracted the attention of many researchers. Since the pioneering work done by Choi [17], various aspects of nanofluid have been discussed. Masuda et al. [18] have examined that the effective thermal conductivity of nano fluids is expected to enhance the heat transfer as compared to conventional heat transfer. Some recent studies of nano fluid due to stretching sheet and peristaltic motion are given in Refs. [19–24].

In this paper we have discussed the influence of nanofluid on peristaltic transport of a hyperbolic tangent fluid model under the effects of inclined magnetic field. The paper is arranged as: The mathematical formulation of the present problem is given in Section 2. In Section 3, the analytical solution of the proposed problem is computed with the help of homotopy perturbation and regular perturbation technique. The graphical results of the present problem are defined in Section 4.

2. Mathematical formulation

We consider the peristaltic transport of an incompressible non-Newtonian fluid (hyperbolic tangent model) in a two dimensional channel of width d_1+d_2, under the effects of apt magnetic field. The channel asymmetry is produced due to different amplitudes and phases of the peristaltic waves. Heat transfer along with nano particle phenomena has been taken into description. The lower wall of the channel is sustained at temperature T_1 and nano particle volume fraction C_1 while the upper wall has temperature T_0 and nano particle volume fraction C_0

The geometry of the wall surface is defined as

$$Y=H_1=d_1+a_1\cos\left[\frac{2\pi}{\lambda}(X-ct)\right],\ Y=H_2=-d_2-b_1\cos\left[\frac{2\pi}{\lambda}(X-ct)+\phi\right], \tag{1}$$

where a_1 and b_1 are the amplitudes of the waves, λ is the wave length, d_1+d_2 is the width of the channel, c is the velocity of

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propagation, t is the time and X is the direction of wave propagation, the phase difference ϕ varies in the range $0 \le \phi \le \pi$, $\phi = 0$ corresponds to symmetric channel with waves out of phase and $\phi = \pi$ the waves are in phase, and further a_1, b_1, d_1, d_2 and ϕ satisfies the condition

$$a_1^2 + b_1^2 + 2a_1 b_1 \cos\phi \le (d_1 + d_2)^2.$$

The governing equations for an incompressible nanofluid under the effect of inclined magnetic field are given by [16,22]

$$\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0, \tag{2}$$

$$\rho_f\left(\frac{\partial U}{\partial t} + U\frac{\partial U}{\partial X} + V\frac{\partial U}{\partial Y}\right) = -\frac{\partial P}{\partial X} + \frac{\partial}{\partial X}(S_{XX}) + \frac{\partial}{\partial Y}(S_{XY}) - \sigma B_0^2 \cos\Theta(U\cos\Theta - V\sin\Theta) + \rho g\alpha(T - T_0) + \rho g\alpha(C - C_0), \tag{3}$$

$$\rho_f\left(\frac{\partial V}{\partial t} + U\frac{\partial V}{\partial X} + V\frac{\partial V}{\partial Y}\right) = -\frac{\partial P}{\partial Y} + \frac{\partial}{\partial X}(S_{YX}) + \frac{\partial}{\partial Y}(S_{YY}) + \sigma B_0^2 \sin\Theta(U\cos\Theta - V\sin\Theta), \tag{4}$$

$$\left(\frac{\partial T}{\partial t} + U\frac{\partial T}{\partial X} + V\frac{\partial T}{\partial Y}\right) = \alpha\left(\frac{\partial^2 T}{\partial X^2} + \frac{\partial^2 T}{\partial Y^2}\right) + \tau\left\{D_B\left(\frac{\partial C}{\partial X}\frac{\partial T}{\partial X} + \frac{\partial C}{\partial Y}\frac{\partial T}{\partial Y}\right)\left(\frac{D_T}{T_0}\right) \times \left[\left(\frac{\partial T}{\partial X}\right)^2 + \left(\frac{\partial T}{\partial Y}\right)^2\right]\right\}, \tag{5}$$

$$\left(\frac{\partial C}{\partial t} + U\frac{\partial C}{\partial X} + V\frac{\partial C}{\partial Y}\right) = D_B\left(\frac{\partial^2 C}{\partial X^2} + \frac{\partial^2 C}{\partial Y^2}\right) + \left(\frac{D_T}{T_0}\right)\left(\frac{\partial^2 T}{\partial X^2} + \frac{\partial^2 T}{\partial Y^2}\right), \tag{6}$$

where U, V are the velocities in X and Y directions in fixed frame, ρ_f is density of fluid base, P is the pressure, ν is the kinematic viscosity, is the temperature, D_B is the Brownian diffusion coefficient, D_T is the thermophoretic diffusion coefficient, $\tau = ((\rho c)_p/(\rho c)_f)$ is the ratio of the effective heat capacity of the nanoparticle material and heat capacity of the fluid with ρ being the density, c is the volumetric volume expansion coefficient and ρ_p is the density of the particles.

The constitutive equation for hyperbolic tangent fluid is given by [7]

$$S = -[[\eta_\infty + (\eta_0 + \eta_\infty)\tan h(\Gamma\dot{\gamma})^n]\dot{\gamma}], \tag{7}$$

in which S is the extra stress tensor, η_∞ is the infinite shear rate viscosity, η_0 is the zero shear rate viscosity, Γ is the time constant, n is the power law index and $\bar{\gamma}$ is defined as

$$\dot{\gamma} = \sqrt{\frac{1}{2}\sum_i\sum_j \dot{\gamma}_{ij}\dot{\gamma}_{ji}} = \sqrt{\frac{1}{2}\Pi}, \tag{8}$$

where

$$\Pi = trac(gradV + (gradV)^T)^2$$

here Π is the second invariant strain tensor. We consider the constitution Eq. (7), the case for which $\eta_\infty = 0$ and $\Gamma\bar{\dot{\gamma}} < 1$. The component of extra stress tensor therefore, can be written as

$$S = -\eta_0[(\Gamma\dot{\gamma})^n]\,\dot{\gamma} = -\eta_0[(1 + \Gamma\dot{\gamma} - 1)^n]\dot{\gamma} = -\eta_0[1 + n(\Gamma\dot{\gamma} - 1)]\dot{\gamma}. \tag{9}$$

The coordinate frames are related by the following transformation

$$x = X - ct, y = Y, u = U - c, v = V, \text{ and } p(x) = P(X, t). \tag{10}$$

Defining the following non-dimensional quantities

$$\bar{x} = \frac{x}{\lambda},\ \bar{y} = \frac{y}{d_1},\ \bar{u} = \frac{u}{c},\ \bar{v} = \frac{v}{c},\ \delta = \frac{d_1}{\lambda},\ d = \frac{d_2}{d_1},\ \bar{p} = \frac{d_1^2 p}{\eta_0 c\lambda},\ \bar{t} = \frac{ct}{\lambda},\ h_1 = \frac{H_1}{d_1},$$

$$h_2 = \frac{H_2}{d_2},\ a = \frac{a_1}{d_1},\ b = \frac{b_1}{d_1},\ R_e = \frac{cd_1}{v},\ \overline{\Psi} = \frac{\Psi}{cd_1},\ \theta = \frac{T - T_0}{T_1 - T_0},\ \bar{S}_{\overline{xx}} = \frac{\lambda}{\eta_0 c}S_{xx},$$

$$\bar{S}_{\overline{xy}} = \frac{d_1}{\eta_0 c}S_{xy},\ \bar{S}_{\overline{yy}} = \frac{d_1}{\eta_0 c}S_{yy},\ We = \frac{\Gamma c}{d_1},\ \bar{\gamma} = \frac{\dot{\gamma}d_1}{c},\ M = \sqrt{\frac{\sigma}{\upsilon}}B_0 d_1,\ \alpha = \frac{K'}{c_p\rho},$$

$$\Pr = \frac{\nu}{\alpha},\ N_T = \frac{\tau D_T(T_1 - T_0)}{T_0\nu},\ N_b = \frac{\tau D_B(C_1 - C_0)}{\upsilon},\ Gr = \frac{\rho g\alpha d_1^2(T_1 - T_0)}{\eta_0 c},$$

$$Br = \frac{\rho g\alpha d_1^2(C_1 - C_0)}{\eta_0 c},\ Le = \frac{\upsilon}{D_B}. \tag{11}$$

Using Eqs. (10) and (11) the resulting equations in terms of stream function Ψ (dropping the bars, $u = (\partial\Psi/\partial y)$, $v = -\delta(\partial\Psi/\partial x)$) can be written as;

$$Re\,\delta(\Psi_y\Psi_{xy} - \Psi_x\Psi_{yy}) = -\frac{\partial p}{\partial x} + \delta\frac{\partial}{\partial x}(S_{xx}) + \frac{\partial}{\partial y}(S_{xy}) + Gr\theta + B_r\Phi - M^2\cos\Theta((\Psi_y + 1)\cos\Theta + \delta\Psi_x\sin\Theta), \tag{12}$$

$$Re\,\delta^3(-\Psi_y\Psi_{xx} + \Psi_x\Psi_{xy}) = -\frac{\partial p}{\partial y} + \delta^2\frac{\partial}{\partial x}(S_{yx}) + \delta\frac{\partial}{\partial y}(S_{yy}) + M^2\delta\sin\Theta((\Psi_y + 1)\cos\Theta + \delta\Psi_x\sin\Theta), \tag{13}$$

$$Re\,\delta(\Psi_y\theta_x - \Psi_x\theta_y) = \frac{1}{\Pr}(\theta_{yy} + \delta^2\theta_{xx}) + N_b(\delta^2\theta_x\Phi_x + \theta_y\Phi_y) + N_T(\delta^2(\theta_x)^2 + (\theta_y)^2), \tag{14}$$

$$Re\,\delta\,Le(\Psi_y\Phi_x - \Psi_x\Phi_y) = (\Phi_{yy} + \delta^2\Phi_{xx}) + \delta^2\frac{N_T}{N_b}\theta_{xx} + \frac{N_T}{N_b}\theta_{yy}, \tag{15}$$

where

$$\begin{aligned} S_{xx} &= 2[1 + n(We\dot{\gamma} - 1)]\frac{\partial^2\Psi}{\partial x\partial y}, \\ S_{xy} &= [1 + n(We\dot{\gamma} - 1)]\left(\frac{\partial^2\Psi}{\partial y^2} - \delta^2\frac{\partial^2\Psi}{\partial x^2}\right), \\ S_{yy} &= -2\delta[1 + n(We\dot{\gamma} - 1)]\frac{\partial^2\Psi}{\partial x\partial y}, \\ \dot{\gamma} &= \left[2\delta^2\left(\frac{\partial^2\Psi}{\partial x\partial y}\right)^2 + \left(\frac{\partial^2\Psi}{\partial y^2} - \delta^2\frac{\partial^2\Psi}{\partial x^2}\right)^2 + 2\delta^2\left(\frac{\partial^2\Psi}{\partial x\partial y}\right)^2\right]^{1/2}, \end{aligned} \tag{16}$$

The corresponding boundary conditions in terms of stream function are defined as

$$\begin{aligned} \Psi &= \frac{q}{2} \text{ at } y = h_1 = 1 + a\cos 2\pi x, \\ \Psi &= -\frac{q}{2} \text{ at } y = h_2 = -d - b\cos(2\pi x + \phi), \\ \frac{\partial\Psi}{\partial y} &= -1 \text{ at } y = h_1 \text{ and } y = h_2, \end{aligned} \tag{17}$$

$$\begin{aligned} \theta &= 0 \text{ on } y = h_1, \\ \theta &= 1 \text{ on } y = h_2, \end{aligned} \tag{18}$$

$$\begin{aligned} \Phi &= 0 \text{ on } y = h_1, \\ \Phi &= 1 \text{ on } y = h_2. \end{aligned} \tag{19}$$

where q is the flux in the wave frame, a, b, ϕ and d satisfy the relation

$$a^2 + b^2 + 2ab\cos\phi \le (1 + d)^2.$$

Under the assumption of long wave length $\delta \ll 1$ and low Reynolds number, Eqs. (12)–(16) become

$$0 = -\frac{\partial p}{\partial x} + \frac{\partial}{\partial y}\left[\left(1 + n\left(We\frac{\partial^2\Psi}{\partial y^2} - 1\right)\right)\frac{\partial^2\Psi}{\partial y^2}\right] - M^2\cos^2\Theta(\Psi_y + 1) + Gr\theta + B_r\Phi, \tag{20}$$